

Family Policies: What Does The Standard Endogenous Fertility Model Tell Us?

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Abstract

Very few studies have explored the optimality properties of the "standard model" of fertility where parents must determine their optimal trade-off between quality and quantity. The present paper works to fill that gap and find three main results. First, when there exist positive externalities in the accumulation of human capital, it is optimal to subsidize education *and* to tax births. Second, when the Social Welfare Function does not consist of the average utility, the social returns on educational investments can be weaker than the private returns when the optimal population growth rate is negative. In this case, the optimal economic policy consists in subsidizing births and taxing education. Finally, when the health expenditure is introduced as another source of positive externalities, it can be optimal to tax the parental health expenditure to decentralize the first-best path even if this expenditure is always too low at the laissez-faire equilibrium.

JEL Codes: H21, I0, J13, J18

Key Words: Fertility, Education, Family Policy, Mortality, Quality Quantity Trade-off

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1 Introduction

The economic analysis of fertility entered the modern era with the contribution of Becker [1960]. This contribution paved the way for analyzing fertility behaviors with the tools of the Marginalist Revolution. Becker [1960], Becker & Tomes [1973] and Becker & Lewis [1976] represent fertility at the family level as the result of a rational decision-making process. In addition to the usual commodities that were already present in microeconomic theory, parents value both the quantity of children (the number of children they give birth to) and their quality. Parents therefore must determine their optimal trade-off between quality and quantity. In recent years, the Unified Growth Theory has put such a trade-off between quality and quantity at the heart of the explanation of long-run growth and development.²

All these contributions have resulted in a unified framework that I here call the "standard model of endogenous fertility." Surprisingly, very few studies have explored the optimality properties of the trade-off between quality and quantity in this model.³ The present paper works to fill that gap, especially by emphasizing that the standard model of fertility displays non-intuitive optimality properties and provides unusual recommendations for economic policy.⁴

The standard model of endogenous fertility displays three main characteristics. First, parents value the number of their offspring (quantity) as well as their future quality. The valuation of children's quality can take either an altruistic or a non-altruistic form. When parents are altruistic toward their children as in Becker & Barro [1988] and Razin & Ben Zion [1975], the future well-being of their children enter their own utility function. Thus, their preferences are dynastic. In the alternative representation, as presented in Becker & Lewis [1973] and Galor & Weil [1999], parents are not altruistic, but are characterized by a joy of giving or a warming glow; that is, they directly value their children's human capital, their wealth, their health status, the financial bequest they give them, or other properties inherent to the children. This representation of parental preferences has been

²Among many others, see Galor & Weil [1996,1999], Galor & Moav [2002], De la Croix & Doepke [2003], Kalemli-Ozcan [2003].

³I provide a review of this literature in section 2.

⁴Notice that, in this paper, family policies are limited to tax - transfer policies. In reality, family policies include a large set of instruments like, for instance, coercive policies and preventive actions.

favored by the recent Unified Growth Theory (UGT hereafter).⁵ Second, whatever the representation that is chosen, parents maximize their expected utility subject to a non-linear budget constraint.⁶ There is therefore a trade-off between quality and quantity. Third, at least in the recent literature, models explicitly assume the existence of inefficiencies in the production of child quality (see Galor [2005]). Quality of children is almost always represented by their human capital. In other words, when parents choose their optimal trade-off between quality and quantity, they do not internalize that their private investment will improve the overall efficiency of the human capital accumulation process. It implies that, at the laissez-faire equilibrium, their arbitrage between quality and quantity cannot be optimal.

Intuitively, because only positive externalities exist in the accumulation of human capital, one can expect that a subsidy on education spending financed by a lump-sum transfer will decentralize the social optimum. In the present paper, I show that this intuition is not precisely correct. I demonstrate this through three important findings:

First, the first-best social optimum cannot be decentralized with less than two Pigouvian taxes⁷ and one lump-sum transfer. In the case where no difference exists between social and individual welfare functions (Millian Social Welfare Function),⁸ these Pigouvian taxes consist of a subsidy to education expenditure and a tax on births. Such a result comes from the parental budget constraint, where quality and quantity enter multiplicatively. This non-linearity implies that distorting the cost of quality to correct human capital externalities distorts, in turn, the total cost of quantity: children become cheaper. A tax on child births must be implemented to correct this second distortion. In other words, though fertility is not a source of externalities, it has to be taxed. This result is robust to changes in the model of fertility that is chosen.

Second, I show that when the Millian Social Welfare Function (SWF hereafter) is no

⁵See Galor [2005] for a review of this literature.

⁶This non-linearity is fundamental in models of trade-off between quality and quantity. Because quality is provided to each child (with or without equity), its cost crucially depends on the quantity choices. Then the parental budget constraint is no longer linear.

⁷Notice that I only focus on linear taxation. Without inequalities, it is not a strong assumption because redistribution is not a matter of concern. Fan & Stark [2008] consider the impact of heterogeneity on welfare and policy analyses.

⁸A Millian Social Welfare Function consists of the average utility in the economy while a Benthamite Social Welfare Function consists of the total utility in the economy.

longer used, the optimal tax-transfer policy can involve taxing education and subsidizing births. I construct a SWF that allows for the existence of a social preference for the population stock. The Benthamite utility function consists in a special case of this SWF.⁹ The existence of a social preference for the population stock introduces two additional effects: (i) Agents do not take into account that, when they make a child, they make it easier for future generations to reach a larger population size. In other words, they do not internalize the social returns on their investment in the quantity of children. (ii) If there exists a social preference for the population stock, there also exists a preference for the largest generations relative to the smallest ones. Thus, the social return on the investment in human capital of one's generation will depend on its size relative to previous and subsequent ones. Formally, it is optimal to transfer welfare from smaller to bigger generations. Thus, when the optimal population growth rate is positive, all other things being equal, the social return on the educational investment is higher than the private return. Indeed, this investment will benefit a growing number of agents, so it is optimal to subsidize education spending. Conversely, if the optimal population growth rate is negative, the largest generations are the current generations and it is optimal to disincentivize parents to invest in their children's human capital in order to transfer utility from future to present generations. In other words, *ceteris paribus*, the social return on education investment is lower than the private return, and it is optimal to tax education.

My third main result comes from the introduction of endogenous child mortality that is a natural extension of endogenous fertility models to address for instance the issue of the demographic transition. It changes the nature of the trade-off between quality and quantity. Indeed, parents not only have to decide how to allocate their spending between quality and quantity, they also have to decide their optimal strategy to reach their desired number of children. In other words, they face a trade-off between quality and quantity of surviving children in which their health expenditure will be a source of externalities.

In this extended model, higher parental health expenditure reduces child mortality. Furthermore, the average level of health spending has a negative impact on child mortality. The literature of development economics provides strong evidence that overall health quality is

⁹Blackorby [2006] provides an enlightening discussion on the caveats of both the Millian and Benthamite SWF.

one of the main determinants of individual health quality. For instance, Dasgupta [1993] shows that 45 percent of all deaths in developing countries can be imputed to infectious and parasitic diseases. Private health expenditure helps reduce the probability of being infected when an agent is in contact with disease. Therefore, a higher average level of health expenditure reduces the probability of death in all families. This positive externality implies that the private health expenditure is too low at the competitive equilibrium.

Here, I only consider the Millian case for simplicity. Reaching optimality requires, once again, subsidizing education and taxing births. Now, the taxation of births plays the role of an indirect subsidy on health expenditure. Indeed, it increases the cost of quantity relative to the cost of health. To reach the same number of surviving children, parents tend to increase their health expenditure and to give birth to less children. For strong externalities on health expenditure, the indirect subsidy will not be sufficient to reach optimal health expenditure at the competitive equilibrium. Therefore, the private health expenditure has to be subsidized.¹⁰ Conversely, if the externality on health is weak relative to that on education (small indirect subsidy), it is optimal to tax health expenditures in addition to births.

The recommendation to tax births in complement to subsidies for education and health, can be analyzed in the light of some empirical evidence from China and Sub-Saharan Africa. Both regions face a problem of overpopulation and have implemented alternative strategies to reduce fertility. My paper's results are obviously theoretical and cannot reproduce the very complex demographic, economic and political conditions of these countries. However, it provides some bases from which their strategies can be called into question.

China is experimenting with a specific fiscal scheme on births that subsidizes the first birth and strongly taxes subsequent ones. However, empirical studies such as those of Kanbur & Zhang [2003] and Fan & Zhang [2000] show that investment in education and health is insufficient in China. The present paper proposes an alternative fiscal scheme that would reallocate public funds from the first birth subsidy to the promotion of education and health, without a loss of efficiency in birth control. However, a polemical interpretation of my results indicate that, if the Chinese optimal fertility rate is closed but inferior to two children per

¹⁰Once again, I focus on linear taxation. I do not deal with health as a pure public good. Doing so would make less plausible that taxing health expenditure is optimal.

family, then the Chinese government policy that consists in taxing births and providing weak public spending on education is optimal.

Sub-Saharan African countries have implemented several family planning programs that strongly promote investment in health and education. However, a recent report from the World Bank [2007] shows that these programs have been inefficient in reducing the net fertility rate in a large majority of these countries. This paper demonstrates that one reason why these policies have been inefficient could lie in the fact that they did not increase the relative cost of quantity. It shows that more attention should be paid to the implementation of a fiscal scheme that would explicitly sanction births.

The rest of the paper is organized as follows. In Section 2, I present the contributions of the present paper to the existing literature. In section 3, the benchmark model is presented. Its recommendations in terms of family policies are discussed. I show that my main results are robust to the adoption of alternative standard models of endogenous fertility. In section 4, I introduce endogenous child mortality and public health. Section 5 discusses the paper's empirical implications for China and Sub-Saharan Africa, and section 6 provides concluding remarks.

2 Contribution to the Existing Literature

My aim is to determine the tax-transfer policy that will decentralize the first-best social optimum. Such an exercise is very typical in the public economics literature, and it consists of determining the distortions that ensure that decentralized individual decisions will lead to the social optimum. In the present framework, I apply this standard methodology to a non-standard problem. Indeed, quality and quantity of children are special goods that cannot be exchanged on a market.^{11,12}

¹¹Following Boulding [1964], a recent paper by De la Croix and Gosseries [2007] relaxes this assumption by assuming the existence of a market of procreation rights that can be exchanged. It finally consists in a system of tax or allowance on the quantity of children. They do not investigate, however, the reasons why governments are not satisfied with their national fertility. Then, the present paper can be considered as a complement to this literature.

¹²I assume that the government can observe the agent's behaviors and expectations, it allows to decentralize the first best social optimum. This assumption is strong but fundamental because it shows that the standard problem of trade-off between quality and quantity (externalities on human capital and non linearity of the parental budget constraint) structurally generates non-optimal *laissez faire* equilibria *regarding both*

Eckstein and Wolpin [1985] paved the way in exploring the optimality of fertility behaviors in a model where parents face a trade-off between quantity and quality. They assume a non-dynastic utility function, wherein parents value the number of their children and their future consumption. Considering Diamond's growth model with endogenous fertility, they show that, at the laissez-faire equilibrium, stationary fertility is lower than the fertility rate that maximizes welfare at the steady state. This result arises because parents do not take into account the impact of their fertility on the stationary interest rate.

Another foundational paper comes from Nerlove *et al* [1982], who explore the optimality of fertility rates in an economy where parents are altruistic and characterized by perfect foresight. They show that the choice of different SWF lead to an alternative judgment about the competitive fertility rate. Indeed, using a Benthamite SWF implies that the competitive fertility rate is lower than optimal while using a Millian SWF can imply a competitive fertility rate that is either lower or higher than optimal. Finally, the Benthamite SWF always lead to a higher optimal fertility rate than the Millian SWF.^{13,14}

The present paper extends the contributions of these earlier papers by focusing on the maximization of welfare across dynamics instead of only at a steady state. Then, I then determine an economic policy that decentralizes the first-best path. Both Eckstein & Wolpin and Nerlove *et al*, conclude that competitive fertility has to be corrected by economic policies because of the existence of externalities on fertility choices. In this paper, even if there is no externality on quantity, the existence of externalities on human capital accumulation (quality of children) implies that it is optimal to tax or subsidize births in addition to subsidizing education. I also show that this result remains valid in both altruistic and non-altruistic models.

Golosov *et al* [2007] explore the optimality of fertility rates in a Barro-Becker model [1988] using their notions of A and P efficiency that I will present in more detail in the core

education and fertility choices. Adding alternative externalities on fertility or studying second order optima will interestingly contrast this result.

¹³Spiegel [1993] extends the Nerlove *et al*'s framework to Rawlsian social preferences. He shows that a poll tax on births enables the government to decentralize the social optimum of the economy.

¹⁴An alternative literature explores the problem of optimal fertility rates with models of endogenous fertility where the quality of children is exogenous. See for instance, Groezen *et al*. [2003] and Loupias & Wigniolle [2004]. Another recent literature is interested in the determination of optimal family policies in a framework where there exist some constraints on the feasible set of economic policy. See for instance, Balestrino *et al* [2000] and Cigno & Pettini [2002].

of the paper. They show that, when there exist external effects that are confined inside the family, perfect altruism implies that "the time series of populations [...] is optimal." It has to be noticed that such a result would be different if the set of externalities that is explored were to include technological externalities, such as learning by doing. In the present paper, the existence of dynastic altruism will not prevent the competitive population growth rate to differ from the optimal one because external effects are not confined to the family.

3 The Benchmark model

Since the seminal approach from Becker *et al* [1973,1974,1988], standard models of fertility assume that parents maximize their utility depending on their own consumption, the quantity of their children and their quality, subject to a non-linear budget constraint. Authors have proposed a variety of methods to model this problem. In this section, I focus on the model of Razin & Ben-Zion [1975] where children's education is provided inside the family and parental utility is dynastic. As shown by Challier & Michel [1996] and Nerlove & Rault [1997], the model of Razin & Ben-Zion provides better results than that of Barro & Becker [1988]. In Section 3, I will show that my fundamental results are validated for alternative utility functions (especially for the Rational Altruism of Barro & Becker and non-dynastic preferences as used in Unified Growth models). I will also show that when education is not provided by parents, but by teachers, as in the work of De la Croix & Doepke [2003], education becomes a source of both positive and negative externalities and, as a result, it can be the case that private education spending has to be taxed.

3.1 The Competitive Equilibrium

The model consists of an overlapping generations economy with L_t adult agents who live for two periods: childhood and adulthood. Children receive education from their parent and do not consume commodities. This education investment is denoted e_t and consists of a schooling time directly provided by the parent.¹⁵ It improves the children's future human

¹⁵Providing education is the unique way to transmit wealth to children. There is no financial bequest contrary to Barro & Becker [1988] and Eckstein & Wolpin [1985] for instance.

capital h_{t+1} such that:¹⁶

$$h_{t+1} = l(e_t, h_t, \bar{h}_t), \quad l'_1 > 0, l''_{11} \leq 0, l'_2 > 0, l''_{22} \leq 0, l'_3 > 0, l''_{33} \leq 0, \quad (1)$$

Function l is strictly increasing and concave with respect to all its arguments. This production function of human capital is closed in De la Croix & Doepke [2003]. I assume non-increasing return to scale. There is an intra-family transmission of human capital: the human capital of parents h_t positively influences the future human capital of children. It can be understood as a quality effect of the schooling time. Moreover, I assume the existence of a Lucas-type aggregate externality: the average level of human capital in the population \bar{h}_t has a positive impact on children's future human capital.¹⁷ Thus, parents do not take into account that their children's human capital affects the production function of other people's grandchildren. Notice that, following equation (1), e_t can be expressed as a function of h_t , \bar{h}_t and h_{t+1} such that: $e_t = e(h_{t+1}, h_t, \bar{h}_t)$ and $e'_1 > 0$, $e'_2 < 0$, $e'_3 < 0$.

When a child born at $t - 1$ becomes an adult, he has to choose his consumption level C_t , the number of his children N_t and their education e_t . For simplicity's sake, families are mono-parental. Individual decisions satisfy the following budget constraint:

$$C_t + \left[\frac{\sigma}{\xi} + \phi \right] w_t h_t X_t + \theta w_t h_t \Omega(X_t) \cdot e_t = w_t h_t \quad (2)$$

where $X_t \equiv \xi N_t$ denotes the number of surviving children at the end of period t and $\xi \in]0, 1[$ the fraction of children who survive to age five. ξ is exogenous in this section, but will be endogenized hereafter. There is no uncertainty about the reproductive success of a family.¹⁸ Each child born takes a part $\sigma \in]0, 1[$ of its parent's time endowment that is normalized to one. Moreover, each surviving child consumes an extra part ϕ of this time.¹⁹ So the cost of a surviving child is greater than the cost of a non surviving child. The opportunity cost of quantity is equal to $\left[\frac{\sigma}{\xi} + \phi \right] w_t h_t X_t$, where w_t denotes the wage per unit of efficient work.

¹⁶Notice that for all function $\Gamma(\alpha_1, \alpha_2, \dots, \alpha_n, \dots)$, Γ'_n represents the partial derivative of Γ with regard to α_n .

¹⁷This assumption is in line with Lucas [1988] and De la Croix & Doepke [2003].

¹⁸So, unlike the models of Sah [1991] and Kalemli-Ozcan [2003] which assume uncertainty, parents will not overshoot their number of children to ensure the compliance of their optimal fertility rate. Because child death is assumed to occur before age five, parents can rapidly ensure the replacement of dead children.

¹⁹Note that $\frac{\sigma}{\xi} + \phi < 1$. $\theta > 0$ is a scalar that allows the relative education costs to vary.

This total cost of quantity includes the ineffective costs engaged for non-surviving children. Consequently, it is negatively dependent on the child survival rate.

The cost of one unit of education is not affected by variations in the child mortality rate. Indeed, no educational investment is engaged until a child reaches age five. The total cost of education is concave in X_t ; one unit of education can benefit more than one child. $\theta w_t h_t \Omega(X_t) \cdot e_t$ is the cost of giving e_t units of education to X_t children with $1 \geq \Omega'(X_t) \geq 0$ and $\Omega''(X_t) \leq 0$.²⁰ If education is a pure public good inside the family ($\Omega(X_t) = 1$), providing e_t units of education to one child implies the same cost as providing e_t units to X_t children. If education is a pure private good inside the family ($\Omega(X_t) = X_t$), one unit of education benefits only one child.

The price of the final good is normalized to one. It is produced in quantity Y_t , following linear technology:

$$Y_t = AH_t \quad (3)$$

where A is a productivity factor and H_t is the total amount of human capital in the workforce. At the labor market's equilibrium, H_t is:

$$H_t = \left[1 - \left(\frac{\sigma}{\xi} + \phi \right) X_t - \theta e_t \Omega(X_t) \right] h_t L_t \quad (4)$$

The workforce participation of a parent consists in his remaining time after childbearing and educating their children. Furthermore, as the labor market is competitive, the wage equals the workers' marginal productivity:

$$w_t = A \quad (5)$$

As in Razin & Ben-Zion [1975], the utility of an agent born in $t-1$, with perfect foresight,²¹ is represented as:²²

$$V_t = \max \{ u(C_t, X_t) + \beta V_{t+1} \} \quad (6)$$

²⁰As shown by Willis [1973], because the cost of education are less intensive in parental time, an increase in the parental income implies a substitution of quality to quantity. $\Omega'(X_t) \leq 1$ ensures the existence of scales economies in childrearing.

²¹I assume that agents formulate perfect expectations in order to ease the resolution of the problem. However, assuming alternative expectations would not alter my main results given that the Social Planner can perfectly observe private expectations.

²²Notice that, as mentioned by Nerlove and Rault [1997], this utility function is additively separable and so, consists in a specific case of $V_t = V(C_t, X_t, V_{t+1})$.

where $u(.,.)$ is strictly increasing and concave in its arguments and $\lim_{\Psi_t \rightarrow 0} u'_{\Psi_t} = +\infty$ for $\Psi_t = \{C_t, X_t\}$. V_t denotes the maximal utility of an adult born in $t - 1$. His current utility depends on his own consumption and the number of his surviving children X_t . I assume that parents value the number of surviving children and not the number of children born. This implies that child mortality is a source of disutility. Parental altruism is dynastic, each agent values his children's discounted welfare per capita. β denotes the parental discount rate. I assume a dynastic utility function because it equalizes the maximization horizon of both individuals and the Social Planner. If I had chosen to assume non-altruistic preferences like in Becker & Tomes or in Unified Growth models, I would have to either focus on the stationary solutions to the problem or to assume the existence of an ad hoc social discount rate introducing artificial dynamics inefficiencies. I discuss this problem in the next section.

Iterating (6) to $t = +\infty$, I obtain the equivalence with the centralized problem²³ where the objective can be written as the following:

$$\max_{C_s, X_s, e_s} V_t = \sum_{s=t}^{+\infty} \beta^{s-t} u(C_s, X_s) \quad (7)$$

A parent born in $t - 1$ determines his optimal demands (C_t^*, X_t^*, e_t^*) by maximizing V_t with respect to L_{t+1} and h_{t+1} ²⁴ subject to (1),(2) and the definition $\frac{L_{t+1}}{L_t} = X_t$. I obtain the following first order conditions with respect to L_{t+1} and h_{t+1} ²⁵:

$$\frac{u'_{X_t} - Ah_t^* \left(\frac{\sigma}{\xi} + \phi + \theta \Omega'_{X_t} e[h_{t+1}^*, \cdot, \cdot] \right) u'_{C_t}}{Ah_{t+1}^* \left(\frac{\sigma}{\xi} + \phi + \theta \Omega'_{X_{t+1}} e[h_{t+2}^*, \cdot, \cdot] \right) u'_{C_{t+1}} - u'_{X_{t+1}}} = -\beta \frac{X_{t+1}^*}{X_t^*} \quad (8)$$

$$\frac{u'_{C_{t+1}}}{u'_{C_t}} = \frac{\theta h_t^* \Omega(X_t^*) e'_1[h_{t+1}^*, \cdot, \cdot]}{\beta [1 - (\frac{\sigma}{\xi} + \phi) X_{t+1}^* - \theta \Omega(X_{t+1}^*) (e[h_{t+2}^*, \cdot, \cdot] + h_{t+1}^* e'_2[h_{t+2}^*, \cdot, \cdot])]} \quad (9)$$

Notice that, ex post, at the equilibrium of the labor market, $\bar{h}_t^* = h_t^*$. By assumption,

²³A necessary condition to obtain such an equivalence is: $\lim_{T \rightarrow +\infty} \beta^T U(C_T, X_T) = 0$. X_t being the number of surviving children, it is bounded by $\frac{1}{\frac{\sigma}{\xi} + \phi}$ the maximal number of children a wife can give birth to. So, this condition will always be satisfied when income and so consumption are bounded at the steady state. If the economy reaches a balanced growth path where consumption grows at a constant rate, the previous condition has to be assumed what is the case for the rest of the paper.

²⁴Note that, h_{t+1} depends on the family's human capital, the average human capital and the educational choices of parents. As parents know the level of h_t and \bar{h}_t when they determine e_t ; choosing e_t is equivalent to choosing h_{t+1} . The same reasoning does apply to X_t . Then, I solve the problem by maximizing with respect to state variables as proposed by Challier & Michel [1996].

²⁵To ensure global concavity of the problem, its Hessian Matrix is assumed to be negative semi-definite.

there is no inequality of human capital. The competitive equilibrium is described by the set $\{C_t^*, X_t^*, e_t^*, h_t^*, h_{t+1}^*, H_t^*, Y_t^*, w_t^*\}_{t=0}^{t=+\infty}$, satisfying equations $\{(1) - (5), (8), (9)\}_{t=0}^{t=+\infty}$. The presence of externalities makes private choices on education inefficient. Parents do not consider the positive effect of their educational investment on the overall efficiency of human capital accumulation. It follows that the competitive equilibrium cannot correspond to the social optimum. The next sub-sections derive the social optimum of the economy and compare it to the competitive equilibrium.

3.2 The Social Optimum

Defining the social optimum when the size of population is endogenous demands a discussion of two concepts: optimality and social welfare. When the size of the population is endogenous, comparing two equilibria requires comparing two situations in which the number of agents is different. This makes the use of the standard concept of Pareto efficiency inadequate. Golosov *et al* [2007] propose the *A* and *P* efficiency concepts. With the *A*-efficiency concept, one equilibrium dominates another if it is preferred by all the agents who live in the two equilibria. With the *P*-efficiency concept, agents who are not born are characterized by well-defined preferences, so an equilibrium dominates if it is preferred by all the agents who could be born in the two equilibria. In other words, agents who are not born in the dominant equilibrium but would be born in the other effectively prefer not to be born.²⁶ As shown by Golosov *et al*, following the concept of efficiency that is chosen, the ranking of equilibria can differ.²⁷ In this paper, I focus on maximizing the utility of current generations but I will show that social optimums do not necessary dominate competitive equilibria in the sense of *A*-efficiency.

As mentioned in the introduction, another critical point lies in the choice of a welfare

²⁶Blackorby *et al* [2005] investigate the concept of critical levels of utility which, if "enjoyed by an added person without changing the utilities of the existing population, leads to an alternative which is as good as the original". One major issue of this literature lies in the choice of critical levels.

²⁷Michel & Wigniolle [2008] propose the concepts of *RC*-Efficiency and *CRC*-Efficiency that refine *A*-Efficiency. With the *RC*-Efficiency concept, an equilibrium dominates another if it is not possible to improve the situation of a generation without reducing the utility of at least another generation what is equivalent to *A*-Efficiency with homogenous agents. With the *CRC*-Efficiency, an equilibrium dominates another if it improves the utility of one generation without reducing the utility and the size of other generations. Conde-Ruiz *et al* [2004] propose the concept *u*-Efficiency that adapts *P*-efficiency to models where number of children is continuous.

function. In the literature, two functions are usually opposed: the Millian SWF corresponding to the utility of the representative agent,²⁸ and the Benthamite SWF corresponding to the total utility in the economy.²⁹ In this paper, I propose a formulation of the SWF that embodies these two cases and intermediary ones. Let W_0 denote the SWF:

$$W_0 = \sum_{t=0}^{+\infty} \beta^t f(L_t) u(C_t, X_t) \quad (10)$$

where $f'(L_t) \geq 0$. $f(L_t)$ denotes the social preference for the population stock. Because $f'(L_t)$ is strictly positive, for a given per capita utility level, the Social Planner prefers larger populations. Notice that, when $f(L_t) = L_t$, the Social Welfare function is Benthamite, whereas when $f(L_t) = 1$, it is Millian. For tractability of results, I assume that $\frac{f(\frac{L}{\eta})}{f(\eta)} = F\left(\frac{L}{\eta}\right)$ with $F'\left(\frac{L}{\eta}\right) > 0$, $F(1) = 1$ and $F(\cdot)$ being twice differentiable.³⁰ To ensure that the Social Welfare Function (SWF) is bounded, I assume $\lim_{T \rightarrow +\infty} \beta^T f(L_T) = 0$.³¹ I also assume that there is no asymmetric information such that the Social Planner can observe all parental preferences, constraints, abilities and expectations.

The resource constraint of the economy implies that, at each date, total production equals total consumption such that:

$$C_t L_t = A h_t \left(1 - \left[\frac{\sigma}{\xi} + \phi \right] X_t + \theta \Omega(X_t) \cdot e_t \right) L_t \quad (11)$$

Then, the Social Planner has to maximize (10) with respect to $\{C_t, X_t, h_{t+1}\}_{t=0}^{t=+\infty}$ and subject to $\{(1), (11)\}_{t=0}^{t=+\infty}$. The social optimum is defined by the set $\{\hat{C}_t, \hat{X}_t, \hat{e}_t, \hat{h}_t, \hat{h}_{t+1},$

²⁸A large set of papers dealing with optimality in endogenous fertility models attribute a Millian objective to the Social Planner. In this representation, the social planner tries to maximize the utility of the representative agent *at the steady state*. See Groezen *et al.* [2003], Loupias & Wigniolle [2004], Zhang [2003], Zhang & Zhang [2007], etc.

²⁹Notice that, contrary to Spiegel [1993], I do not deal with Rawlsian objectives.

³⁰An intuitive formulation of $f(L_t)$ consists in the Cobb-Douglas case such that $f(L_t) = L_t^\kappa$ with $\kappa \in [0, 1]$. If $\kappa = 0$, the objective function is Millian and, if $\kappa = 1$, it is Benthamite.

³¹This condition is equivalent to $\lim_{T \rightarrow +\infty} \beta^T f\left(L_0 \prod_{t=0}^T X_t\right) = 0$. Because fertility is bounded by the maximal number a parent can give birth to, a sufficient condition to ensure that the SWF is bounded is $\lim_{T \rightarrow +\infty} \beta^T f\left(L_0 \left[\phi + \frac{\sigma}{\xi}\right]^{-T}\right) = 0$. It follows that, for a Cobb-Douglas specification such that $f(L_t) = L_t^\kappa$, $\beta < \phi + \frac{\sigma}{\xi}$ is a sufficient condition to ensure that SWF is bounded whatever $\kappa \in [0, 1]$.

$\{\widehat{H}_t, \widehat{Y}_t\}_{t=0}^{t=+\infty}$ satisfying equations $\{(1), (3), (4), (11)\}_{t=0}^{t=+\infty}$ and, at each date t , both following first order conditions with respect to L_{t+1} and h_{t+1} .³²

$$-\frac{1}{\beta} = \frac{\varepsilon_L^f u(\cdot, \widehat{X}_{t+1}) + F(\widehat{X}_t) \frac{\widehat{X}_{t+1}}{\widehat{X}_t} \left[A \widehat{h}_{t+1} \left(\frac{\sigma}{\xi} + \phi + \theta \Omega'_{X_{t+1}} e \left[\widehat{h}_{t+2}, \cdot, \cdot \right] \right) u'_{C_{t+1}} - u'_{X_{t+1}} \right]}{u'_{X_t} - A h_t \left(\frac{\sigma}{\xi} + \phi + \theta \Omega'_{X_t} e \left[\widehat{h}_{t+1}, \cdot, \cdot \right] \right) u'_{C_t}} \quad (12)$$

$$\frac{u'_{C_{t+1}}}{u'_{C_t}} = \frac{\theta h_t \Omega(X_t) e'_1[\widehat{h}_{t+1}, \cdot]}{\beta F(\widehat{X}_t) \left[1 - \left(\frac{\sigma}{\xi} + \phi \right) \widehat{X}_{t+1} - \theta \Omega(\widehat{X}_{t+1}) \left(e[\widehat{h}_{t+2}, \cdot, \cdot] + \widehat{h}_{t+1} (e'_2[\widehat{h}_{t+2}, \cdot, \cdot] + e'_3[\widehat{h}_{t+2}, \cdot, \cdot]) \right) \right]} \quad (13)$$

where $\varepsilon_v^g \equiv \frac{\partial g}{\partial v} \frac{v}{g}$ denotes the elasticity of g with respect to v . Obviously, at the social optimum, all of the existing externalities are taken into account. In this economy, there exist two types of externality: (i) a Lucas-type externality in the accumulation of human capital and (ii) when $f'(L_t)$ is strictly positive, parental preferences differ from the preferences of the Social Planner since parents do not value the population stock.

3.3 The Optimal Tax-Transfer Policy

In order to decentralize the social optimum, the government has to implement a public policy that makes the competitive equilibrium $\{C_t^*, X_t^*, e_t^*, h_t^*, h_{t+1}^*, H_t^*, Y_t^*\}_{t=0}^{t=+\infty}$ coincide with the social optimum $\{\widehat{C}_t, \widehat{X}_t, \widehat{e}_t, \widehat{h}_t, \widehat{h}_{t+1}, \widehat{H}_t, \widehat{Y}_t\}_{t=0}^{t=+\infty}$.

In this section, I discuss the optimal tax-transfer policies in the Millian case where $f'(L_t) = 0$ and in the more general case where $f'(L_t) > 0$. In the Millian case, the only externality is the Lucas-type externality on human capital accumulation that makes parents undervalue the return on their investment in children's human capital. There is no difference between social and private preferences. It therefore seems intuitive that the implementation of a subsidy on education financed by a lump tax would ensure the decentralization of the social optimum. However, one major result of this paper is that this policy cannot decentralize the first-best equilibrium. Indeed, because the budget constraint is not linear, subsidizing education spending reduces the total cost of children, so an additional tax on child births is needed.

In the non-Millian case, in addition to the Lucas-type externality on education, there exists an externality on fertility. The optimal policy will consist of a tax-transfers policy

³²To ensure global concavity of the problem, its Hessian matrix is assumed to be negative semi-definite.

on education and births completed by a lump-sum transfer as in the Millian case, but such a policy does not necessarily consist of taxing births and subsidizing education. The social preference for the population stock enriches the model with two mechanisms. First, when adults decide their number of children, they do not take into account that they make it easier to attain a greater population size in the future, which is socially desirable since it will increase the total utility of society. Second, if $L_{t+1} > L_t$, transferring welfare from current to future generations would be socially desirable because it would benefit a larger population.³³ In other words, the social returns on education investments would be greater than the private ones even if the Lucas-type externality did not exist. However, if $L_{t+1} < L_t$, it is socially desirable to transfer utility from future to present generations. To summarize, the Social Planner prefers transferring utility to the generations with the greatest population. The optimal tax-transfer policy will result from the opposition of the two externalities of the model.

Let the set $\{\lambda_t, \Lambda_t, T_t\}_{t=0}^{t=+\infty}$ define an economic policy where $\lambda_t > 0$ (resp $\lambda_t < 0$) consists of the subsidy rate (resp tax rate) on education spending, $\Lambda_t > 0$ (resp $\Lambda_t < 0$) denotes the tax (resp subsidy) on each birth and $T_t \gtrless 0$ the lump-sum transfer. At each date t , the government budget constraint has to be balanced such that:

$$T_t = \lambda_t \theta e \left(h_{t+1}^*, h_t^*, \bar{h}_t^* \right) \Omega(X_t^*) A h_t^* - \frac{\Lambda_t}{\xi} X_t^* A h_t^* \quad (14)$$

The parental budget constraint then becomes:

$$C_t + \left[\frac{\sigma + \Lambda_t}{\xi} + \phi \right] w_t h_t X_t + (1 - \lambda_t) \theta w_t h_t \Omega(X_t) \cdot e_t = w_t h_t + T_t \quad (15)$$

The competitive equilibrium is now defined by the set $\{C_t^*, X_t^*, e_t^*, h_t^*, h_{t+1}^*, H_t^*, Y_t^*, w_t^*\}_{t=0}^{t=+\infty}$ satisfying equations $\{(1), (3), (4), (5), (15)\}_{t=0}^{t=+\infty}$ and the following first order conditions

³³This mechanism can also be understood in the light of a comparison between private and social preferences for present. Indeed, at period t , equation (7) indicates that in $t = 0$, the per se private welfare gain of increasing by one unit the consumption of each generation is $u'_{C_0} + \beta u'_{C_1} + \beta^2 u'_{C_2} + \dots + \beta^t u'_{C_t} + \dots$ while the per se social welfare gain of this improvement equals $u'_{C_0} + \beta \frac{f(L_1)}{f(L_0)} u'_{C_1} + \beta^2 \frac{f(L_2)}{f(L_0)} u'_{C_2} + \dots + \beta^t \frac{f(L_t)}{f(L_0)} u'_{C_t} + \dots = u'_{C_0} + \beta F(X_1) u'_{C_1} + \beta^2 F(X_1 X_2) u'_{C_2} + \dots + \beta^t F\left(\prod_{s=0}^{s=t} X_s\right) u'_{C_t} + \dots$

It follows from the definition of $F(\cdot)$ that if population is growing ($X_s > 1$) the social discount rate evaluated by $\beta^t F\left(\prod_{s=0}^{s=t} X_s\right)$ is higher than the private one (β^t). Then the social returns of education are higher than the private one even if they would not exist any externality on education investments.

with respect to L_{t+1} and h_{t+1} :

$$-\beta \frac{X_{t+1}}{X_t} = \frac{u'_{X_t} - Ah_t \left(\frac{\sigma + \Lambda_t}{\xi} + \phi + (1 - \lambda_t) \theta \Omega'_{X_t} e[h_{t+1}, \cdot, \cdot] \right) u'_{C_t}}{Ah_{t+1} \left(\frac{\sigma + \Lambda_{t+1}}{\xi} + \phi + (1 - \lambda_{t+1}) \theta \Omega'_{X_{t+1}} e[h_{t+2}, \cdot, \cdot] \right) u'_{C_{t+1}} - u'_{X_{t+1}}} \quad (16)$$

$$\frac{u'_{C_{t+1}}}{u'_{C_t}} = \frac{(1 - \lambda_t) \theta h_t \Omega(X_t) e'_1[h_{t+1}, h_t, \bar{h}_t]}{\beta \left[1 - \left(\frac{\sigma + \Lambda_{t+1}}{\xi} + \phi \right) X_{t+1} - (1 - \lambda_{t+1}) \theta \Omega(X_{t+1}) (e[h_{t+2}, h_{t+1}, \bar{h}_{t+1}] + h_{t+1} e'_2[h_{t+2}, h_{t+1}, \bar{h}_{t+1}]) \right]} \quad (17)$$

Since (14) and (15) ensures that the resources constraint is satisfied, it is straightforward that an optimal economic policy has to equalize subsystems $\{(12), (13)\}_{t=0}^{t=+\infty}$ to $\{(16), (17)\}_{t=0}^{t=+\infty}$.

Proposition 1 *Given the parental (perfect) expectations on $\{\lambda_{t+1}, \Lambda_{t+1}, T_{t+1}\}_{t=0}^{t=+\infty}$, there exists a unique vector $\{\lambda_t, \Lambda_t, T_t\}_{t=0}^{t=+\infty}$ that is able to decentralize the first-best path. Given that \hat{T}_t is directly deduced from the government budget constraint the optimal economic policy is fully described as follows:*

$$\begin{aligned} \hat{\lambda}_t &= -a_t \hat{\lambda}_{t+1} + b_t \hat{\Lambda}_{t+1} + d_t [f(X_t) - 1] - g_t \\ \hat{\Lambda}_t &= -i_t \hat{\lambda}_{t+1} + j_t \hat{\Lambda}_{t+1} - k_t [f(X_t) - 1] - m_t \end{aligned}$$

Proof. See Appendix A. ■

It appears that once the Social Planner observes the parental expectations on the future values of the instruments, it is always possible to define a unique optimal economic policy. At the steady state, the optimal economic policy is described by the set $\left\{ \hat{\lambda}, \frac{\hat{\Lambda}}{\xi} \right\}$:

$$\hat{\lambda} = \frac{-F(X) \varepsilon_3^e - \frac{\beta \varepsilon_L^f X}{Ah \Omega(X) \theta e (1 - \beta F(X))} \frac{u}{u'_C} - [1 - F(X)] \left[\frac{C}{Ah \Omega(X) \theta e} - \varepsilon_2^e \right]}{1 + \frac{\varepsilon_1^e}{\beta} + \varepsilon_2^e - \varepsilon_X^\Omega} \quad (18)$$

$$\frac{\hat{\Lambda}}{\xi} = - \frac{\frac{\beta \varepsilon_L^f}{[1 - \beta F(X)] Ah} \frac{u}{u'_C} \frac{1 + \frac{\varepsilon_1^e}{\beta} + \varepsilon_2^e}{1 + \frac{\varepsilon_1^e}{\beta} + \varepsilon_2^e - \varepsilon_X^\Omega}}{\frac{\varepsilon_1^e}{\beta} + \varepsilon_2^e - \varepsilon_X^\Omega} - \frac{\Omega'_X \theta e \left[F(X) \varepsilon_3^e + (1 - F(X)) \left(\frac{C}{Ah} - \Omega(X) \theta h e'_2 \right) \right]}{1 + \frac{\varepsilon_1^e}{\beta} + \varepsilon_2^e - \varepsilon_X^\Omega} \quad (19)$$

In the following sub-sections, I interpret this result at the steady state in the simple case of a Millian SWF and in the general case where the SWF is not Millian.

3.3.1 The Millian Case: $f'(L_t) = 0$

In the Millian case, there is no difference between individual and social preferences. At the steady state, the optimal economic policy is described as follows:

$$\hat{\lambda} = -\frac{\varepsilon_3^{\hat{e}}}{1 + \frac{\varepsilon_1^{\hat{e}}}{\beta} + \varepsilon_2^{\hat{e}} - \varepsilon_X^{\Omega}} \quad \hat{\frac{\Lambda}{\xi}} = -\frac{\Omega'_X \theta \varepsilon_3^{\hat{e}}}{1 + \frac{\varepsilon_1^{\hat{e}}}{\beta} + \varepsilon_2^{\hat{e}} - \varepsilon_X^{\Omega}} \quad \hat{t} = \frac{A \hat{h} \theta \varepsilon_3^{\hat{e}} (\varepsilon_X^{\Omega} - 1)}{1 + \frac{\varepsilon_1^{\hat{e}}}{\beta} + \varepsilon_2^{\hat{e}} - \varepsilon_X^{\Omega}} \quad (20)$$

The non increasing return to scale in human capital accumulation and $\varepsilon_X^{\Omega} < 1$ implies that $\hat{\lambda} > 0$, $\hat{\frac{\Lambda}{\xi}} > 0$ and $\hat{T} > 0$.³⁴ Because of the Lucas-type externality, parents do not internalize all the returns on their investment in education. Then, at the competitive equilibrium, their investment in children's human capital is smaller than at the optimum. Thus, a subsidy on private education spending has to be implemented. Such a subsidy decreases the marginal cost of education, which becomes $(1 - \lambda)\theta Ah\Omega(X)$ but it also reduces the marginal cost of quantity, which becomes $\left[\frac{\sigma}{\xi} + \phi\right] w_t h_t + (1 - \lambda)\theta w_t h_t \Omega'_{X_t} \cdot e_t$. Thus, parents give births to too many children at the competitive equilibrium, and a tax on each birth has to be implemented such that the marginal cost of quantity becomes $\left[\frac{\sigma+\Lambda}{\xi} + \phi\right] w_t h_t + (1 - \lambda)\theta w_t h_t \Omega'_{X_t} e_t$.³⁵ Therefore, to decentralize the first-best optimum, the government has to implement a tax on births in addition to the education policy despite the fact that the quantity of children is not a source of externalities. In other words, because of the fundamental non-linearity of the quality-quantity costs structure, three instruments are needed to correct the Lucas-type externality.³⁶

Notice that the first-best optimum dominates all other equilibria in the sense of A , RC , CRC -efficiency. Indeed, it is impossible to improve the welfare of one generation of living

³⁴Indeed, non increasing returns to scale in education investment ensure that $\varepsilon_{e_t}^{h_{t+1}} + \varepsilon_{h_t}^{h_{t+1}} + \varepsilon_{\bar{h}_t}^{h_{t+1}} \leq 1$, what implies that $\varepsilon_{e_t}^{h_{t+1}} < 1$ and so that $\varepsilon_{h_{t+1}}^{e_t} = \frac{1}{\varepsilon_{e_t}^{h_{t+1}}} > 1$. By (1) and the definition of elasticity, $\varepsilon_{h_t}^{e_t} = -\left|\varepsilon_{h_t}^{h_{t+1}} \varepsilon_{h_{t+1}}^{e_t}\right|$, so $\frac{\varepsilon_1^{\hat{e}}}{\beta} + \varepsilon_2^{\hat{e}} = \varepsilon_{h_{t+1}}^{e_t} \left[\frac{1}{\beta} - \varepsilon_{h_t}^{h_{t+1}}\right] > 0$. Furthermore, by definition, $\varepsilon_X^{\Omega} < 1$. Then, $1 + \frac{\varepsilon_1^{\hat{e}}}{\beta} + \varepsilon_2^{\hat{e}} - \varepsilon_X^{\Omega} > 0$. As $\varepsilon_3^{\hat{e}} < 0$, it follows that $\hat{\lambda} > 0$, $\hat{\frac{\Lambda}{\xi}} > 0$ and $\hat{T} > 0$.

³⁵As shown by Willis [1973], facing a decrease in the cost of quality, parents can also reduce their fertility rate. In the present case, this can mean, for instance, that the quantity of children is a Giffen good. Indeed, after a decrease in the price of quantity (through the subsidy on education), its "consumption" decreases. Nevertheless, in this case, the optimal economic policy still consists in taxing births to incite parents to make more children.

³⁶When education is a pure public good inside the family ($\Omega'_{X_t} = 0$), taxing births is never necessary to decentralize the first best path. Indeed, education has still to be subsidized but the marginal cost of childbearing does no more depend on the educational investment.

agents without reducing the welfare of another generation of living agents. In that case, the social optimum necessarily dominates the competitive equilibrium. Notice that this result crucially comes from the absence of distance between social and private preferences.

3.3.2 The General Case

When $f'(L_t) > 0$, there exists a difference between individual and social preferences. Indeed, the Social Planner has a preference for the population stock though it is not a concern of individuals. Formally, the optimal tax transfer policy is described by equations (18) and (19) :

$$\hat{\lambda} = \frac{-F(X)\varepsilon_3^e - \frac{\beta\varepsilon_L^f X}{Ah\Omega(X)\theta e(1-\beta F(X))} \frac{u}{u'_C} - [1-F(X)] \left[\frac{C}{Ah\Omega(X)\theta e} - \varepsilon_2^e \right]}{1 + \frac{\varepsilon_1^e}{\beta} + \varepsilon_2^e - \varepsilon_X^\Omega} \equiv \frac{-F(X)\varepsilon_3^e + LHS_\lambda + RHS_\lambda}{1 + \frac{\varepsilon_1^e}{\beta} + \varepsilon_2^e - \varepsilon_X^\Omega} \quad (21)$$

$$\frac{\hat{\Lambda}}{\hat{\xi}} = \frac{\Omega'_X \theta e F(X) \varepsilon_3^e - \beta \varepsilon_L^f \frac{1 + \frac{\varepsilon_1^e}{\beta} + \varepsilon_2^e}{[1-\beta F(X)] Ah} - \Omega'_X \theta e (1-F(X)) \left(\frac{C}{Ah} - \Omega(X) \theta h e'_2 \right)}{1 + \frac{\varepsilon_1^e}{\beta} + \varepsilon_2^e - \varepsilon_X^\Omega} \equiv \frac{\Omega'_X \theta e F(X) \varepsilon_3^e + LHS_\Lambda + RHS_\Lambda}{1 + \frac{\varepsilon_1^e}{\beta} + \varepsilon_2^e - \varepsilon_X^\Omega} \quad (22)$$

The impact of the Lucas-type externality remains the same. The social preference for the size of populations introduces two additional effects. First, the Social Planner prefers the largest generations (for example, distributing one unit of utility per se to a large generation is more enjoyable than distributing one unit of utility to a smaller generation). Thus, when $\hat{X} > 1$ (increasing population), it is optimal to transfer welfare from present to future generations. To do so, educational investments of present generations have to be increased. In other words, the social returns on parental investments in education is higher than the private returns because it will benefit a growing population. This mechanism has a positive effect on the optimal value of the subsidy on education ($RHS_\lambda > 0$). Because the parental budget constraint is non-linear, it has also a positive impact on the optimal tax rate on births ($RHS_\Lambda > 0$). In contrast, if the optimal population growth rate is negative ($\hat{X} < 1$), it is socially enjoyable to transfer welfare from future to present generations, which necessitates limiting the parental investment in education. This has a negative impact on both $\hat{\lambda}$ and $\frac{\hat{\Lambda}}{\hat{\xi}}$ ($(RHS_\lambda, RHS_\Lambda) < (0, 0)$).

The second effect introduced by the existence of a social preference for the size of the population is more straightforward. When deciding their fertility rate, individuals do not

take the social preference for the population stock into account.³⁷ Individuals therefore underestimate the returns on their investment on childbearing. *Ceteris paribus*, their fertility rate is too low, which has a negative impact on both $\hat{\lambda}$ and $\hat{\frac{\Lambda}{\xi}}$ ($(LHS_{\lambda}, LHS_{\Lambda}) < (0, 0)$).³⁸ This mechanism has a negative impact on $\hat{\frac{\Lambda}{\xi}}$ (and on $\hat{\lambda}$ because of the non linearity of the parental budget constraint).

As a result, the optimal tax transfer policy consists of subsidizing education and taxing births as in the Millian case only if the Lucas-type externality is strong relative to the preference for the population stock. This is satisfied when $-\varepsilon_3^e > \max \left\{ \frac{LHS_{\lambda} + RHS_{\lambda}}{F(X)}, \frac{LHS_{\Lambda} + RHS_{\Lambda}}{F(X)} \right\}$.

It has to be noticed that when $f'(L_t) > 0$, the social optimum does not necessarily dominates the laissez-faire equilibrium in the sense of A , RC , CRC , P and u efficiency. Indeed, when the two objectives differ, nothing ensures that all living agents in the two equilibria enjoy a higher welfare at the optimum. Because of the social preference for the population stock, it can be socially enjoyable to give birth to a higher number of agents who receive less welfare than at the laissez-faire (this will be the case when the externality on education investment is weak relative to the social preference for the size of population). Thus, in this sense, the social optimum does not necessarily dominate the laissez-faire equilibrium in the sense of A , RC and CRC -efficiency. Then, because nothing ensures that, for all configuration of the model, the social optimum dominates the competitive equilibrium in the sense of A -efficiency, nothing can ensure such a domination in the sense of P -efficiency.³⁹

In this section, I find two important results. First, in the standard model of endogenous fertility, when there is no difference between private and social preferences, positive externalities on the human capital accumulation process mean that subsidizing education spending is optimal (like in the usual literature) but has to be combined with a tax on each birth. This result comes from the decrease in the marginal cost of quantity induced by education subsidies. Second, when the Social planner is not Millian, despite the fact that education is a source of only positive externalities, subsidizing education is not always optimal. This result comes from the social preference for large generations that distorts the returns to

³⁷ Agents do not take into account that when they give birth to a child, they make it easier for future generations to reach larger population size what is socially enjoyable.

³⁸ It is straightforward that $(LHS_{\lambda}, LHS_{\Lambda}) < (0, 0)$ because, to ensure that the Social Planner's objective is bounded, the model assumes that $\beta F(\max\{X\}) < 1$.

³⁹ In this paper, unborn agents are not characterized by well-defined preferences.

investments in education relative to the returns to investment in the quantity of children.

In the next section, I answer a natural question: "Do the previous results still prevail if I adopt another standard model of fertility?"

4 Alternative "standard" models of endogenous fertility

At least two alternative models could also be considered as standard models of endogenous fertility: the Barro-Becker type model [1988] and the non-altruistic model. I show that adopting these models does not change my main results. I also show that adopting the De la Croix & Doepke's assumption that education is not provided by parents but by teachers can deeply change the nature of the optimal economic policy in the Millian case.

4.1 The Dynastic Altruism of Barro and Becker

Becker & Barro [1988] propose a model of fertility where the parental utility function consists of the discounted sum of their dynasty's flow of utility. Parental altruism is endogenous and negatively related to the quantity of children and the current utility of parents is not influenced by their fertility. In such a case, $V_t = \max \{U(C_t) + \beta(X_t) X_t V_{t+1}\}$ with $\beta(X_t) = \beta X_t^{-\varepsilon}$. In their framework, the utility function of a parent born in $t - 1$ becomes:⁴⁰

$$V_t = \sum_{s=t}^{+\infty} \beta^{s-t} X_s^{1-\varepsilon} U(C_s) \quad (23)$$

where $U(\cdot)$ is strictly increasing and concave in C_s and $\varepsilon \in [0, 1]$. I did not adopt this utility function in my benchmark model. However, as mentioned by Nerlove and Rault [1997], the Barro and Becker's model, as well as the model designed by Razin and Ben Zion, are specific cases of a more general model.⁴¹ Nerlove and Rault [1997] show that the Becker and Barro specification exhibits some unpleasant microeconomic properties, for instance that parental

⁴⁰See Becker and Barro [1988].

⁴¹Nerlove & Rault [1997] present this more general model where $V_t = \max \{u((C_t, X_t) + \beta(X_t) X_t V_{t+1})\}$. In the Becker and Barro specification, $u'_{X_t} = 0$ while in the Razin and Ben Zion model, $X_t \beta(X_t) = \beta$. Notice that in the Becker and Barro specification, when $\varepsilon = 1$, the parental utility function is no more strictly concave with respect to the quantity of children.

fertility responds positively to an increase in income, which is counterfactual. The Razin and Ben Zion version overcomes this problem (see Challer and Michel [1996]).

In the Barro and Becker model, the SWF becomes:

$$W_t = \sum_{s=t}^{+\infty} \beta^{s-t} f(L_s) X_s^{1-\varepsilon} U(C_s)$$

The Barro and Becker model does not introduce any additional externality compared to the model of Razin and Ben Zion. The only difference between the two comes from the formulation of individual utility functions. It is therefore intuitive that the two models lead to the same conclusions in term of optimal economic policy. The proof of this result is provided in Appendix B.

4.2 Non Altruistic Model

Recent literature pertaining to the UGT has favored the use of fertility models where parental utility is not dynastic. In other words, parents exhibit imperfect altruism and the quality of children directly enters their utility function. This quality can take the form of human capital (see Galor [2005]), financial bequest (see Becker and Lewis [1973]), health status, etc. Parental utility can be represented by $U(C_t, X_t, h_{t+1})$, which is increasing and strictly concave with respect to all its arguments. Adopting this non-dynastic representation of preferences makes it difficult to properly define a SWF. In this sub-section, I adopt the method of Eckstein and Wolpin [1985] by defining the SWF as the utility of the current generation at the steady state. I determine the optimal economic policy that allows the decentralization of the set of optimal behaviors at the steady state. Another alternative could consist of defining an ad-hoc social discount rate and assuming a dynastic Social Planner. However, in that case, the model introduces artificial positive externalities on fertility and education choices: parents do not take the dynamic returns on their investments into account .

The stationary Millian SWF is $W = U(C, X, h)$. It is maximized with respect to C, X, h and subject to the stationary resource constraint and the human capital production function

which are, respectively:

$$C = \left[1 - \left(\frac{\sigma}{\xi} + \phi \right) X - \theta \Omega(X) e \right] Ah \quad (24)$$

$$h_{t+1} = l(e_t, h_t, \bar{h}_t) \quad (25)$$

At the stationary competitive equilibrium, agents maximize $V = U(C, X, h)$ with respect to C, X, h and subject to (25) and the following budget constraint:

$$C + \left[\frac{\sigma}{\xi} + \phi \right] whX + \theta wh \Omega(X) \cdot e = wh$$

There is no difference between individual and social preferences and a Lucas-type externality exists, as in the Benchmark model. An additional intradynastic externality is introduced in this model. Indeed, because parents do not care about the future well-being of their children, when they decide their optimal investment in education, they do not take into account that: (i) they increase their children's future earning abilities $w_{t+1}h_{t+1}$, (ii) they reduce the cost of producing the human capital of their grandchildren in the sense that to reach the same $h_{t+2} = l(e_{t+1}, h_{t+1}, \bar{h}_{t+1})$, their children will need to invest a smaller e_{t+1} , and (iii) they increase the opportunity cost of grandchildren relative to the opportunity cost of providing them with education; indeed the quantity of children is more time-consuming than children's quality. The addition of this positive intra-dynastic externality on human capital dramatically reinforces the results of Section 1 in the Millian case.

Formally, the optimal economic policy becomes:⁴²

$$\hat{\lambda} = \frac{\frac{\hat{C}}{Ah} - \theta \Omega(\hat{X}) h (\hat{e}'_2 + \hat{e}'_3)}{\theta \Omega(\hat{X}) \hat{h} \hat{e}'_1}, \quad \frac{\hat{\Lambda}}{\hat{\xi}} = \theta \Omega'(\hat{X}) \hat{e} \frac{\frac{\hat{C}}{Ah} - \theta \Omega(\hat{X}) h (\hat{e}'_2 + \hat{e}'_3)}{\theta \Omega(\hat{X}) \hat{h} \hat{e}'_1} \quad (26)$$

It is straightforward that $(\hat{\lambda}, \frac{\hat{\Lambda}}{\hat{\xi}}) > (0, 0)$. Finally, it appears that considering dynastic or non-dynastic altruism leads to the same fundamental result: in a Millian economy, in the presence of positive externalities in the accumulation of human capital, an optimal economic policy has to subsidize education and has to tax births. However, in the non altruistic model, nothing can ensure that the economic policy that is optimal at the steady state will also be optimal out of the steady state.

⁴²Both the stationary competitive equilibrium and the stationary optimum are displayed in appendix C.

4.3 When education is provided by teachers

De la Croix and Doepke [2003] assume that education is not provided by parents but by teachers. Parents finance a schooling time e_t , the process of accumulation of human capital remains unchanged and the average human capital in the school system is the same as in the whole economy. However, the parental budget constraint and the aggregate human capital in the workforce become respectively:

$$C_t + \left[\frac{\sigma}{\xi} + \phi \right] w_t h_t X_t + \theta w_t \bar{h}_t \Omega(X_t) \cdot e_t = w_t h_t \quad (27)$$

$$H_t = \left[1 - \left(\frac{\sigma}{\xi} + \phi \right) h_t X_t - \theta e_t \Omega(X_t) \bar{h}_t \right] L_t \quad (28)$$

The cost of educating children is no longer an opportunity cost, but a financial cost. The workforce participation of parents consists in their remaining time after childbearing, and teachers do not directly participate in the production of the final good.

For simplicity's sake, I assume that the SWF is Millian. In addition to simplifying the results in a sensible manner, it is intuitive that assuming a non-Millian Social Planner would result in the existence of a preference for larger generations, as in the Benchmark model. Formally, the optimal economic policy at the steady state is now described by:⁴³

$$\hat{\lambda} = \frac{1 + \varepsilon_3^{\hat{e}}}{\varepsilon_2^{\hat{e}} - \frac{\varepsilon_1^{\hat{e}}}{\beta} + \varepsilon_{\hat{X}}^{\Omega}} \quad \hat{\Lambda} = \Omega'_{\hat{X}} \theta \hat{e} \frac{1 + \varepsilon_3^{\hat{e}}}{\varepsilon_2^{\hat{e}} - \frac{\varepsilon_1^{\hat{e}}}{\beta} + \varepsilon_{\hat{X}}^{\Omega}}$$

As in the Benchmark model, parents do not internalize the impact of their investment in human capital at the level of the future average of human capital in the entire population. Thus, they do not consider that, when they invest in their children's human capital, they improve the quality of the school system and so ease the future accumulation of human capital in their dynasty (quality effect). But they also do not take into account that this will in turn increase the cost of future investments in education: indeed, the rise in the average level of teachers' human capital triggers the cost of financing one unit of education (cost effect). This effect is an intra-dynastic negative externality and it can lead to a too high investment in human capital at the competitive equilibrium. The optimal economic policy on education will consist of a tax on education spending when the cost effect is stronger than

⁴³See Appendix D for a complete description of the competitive equilibrium, the first best path and the economic policy that allows to decentralize this latter.

the quality effect ($|\varepsilon_3^{\widehat{e}}| > 1$).⁴⁴ Following De la Croix and Doepke [2003], this condition holds in empirical data. Because no externality exists on fertility, the non-linearity of the parental budget constraint implies that births must be subsidized.

This result highlights that the consideration of the dynamic properties of the standard model of fertility throws into question the common results of education models which do not consider endogenous fertility. It also appears that introducing, in the benchmark model, a mixed education system with both parents and teachers provides richer results. Indeed, even in the Millian case, it can be optimal to tax education and subsidize births despite education being a source of positive externalities.

In the following section, the benchmark model is extended to include endogenous child mortality and private health expenditure. Despite the changes in the nature of the trade-off between quality and quantity, the need to tax births will not be canceled by the introduction of a health expenditure.

5 Optimal Tax-Transfer policy with health expenditure

The child survival rate is currently endogenous. Parents can use health expenditure to reduce their children's probability of dying. In line with Chakraborty [2004], the child survival probability ξ_t is now:

$$\xi_t \equiv \xi(s_t, \bar{s}_t) \quad (29)$$

Parental expenditure on health has a strictly positive and concave influence on children's survival probability, so $\xi'_1 \equiv \frac{\partial \xi(s_t, \bar{s}_t)}{\partial s_t} > 0$ and $\xi''_{11} \equiv \frac{\partial^2 \xi(s_t, \bar{s}_t)}{\partial s_t^2} < 0$. This expenditure represents the health care provided by parents to children. Parental health care covers a large set of expenditures such as hygiene, sanitation improvements and efficient nutrition. \bar{s}_t denotes the average health expenditure in the economy. In line with Dasgupta [1993], $\xi'_2 \equiv \frac{\partial \xi(s_t, \bar{s}_t)}{\partial \bar{s}_t} > 0$ and $\xi''_{22} \equiv \frac{\partial^2 \xi(s_t, \bar{s}_t)}{\partial \bar{s}_t^2} < 0$.

The introduction of an externality on health expenditure implies that the parental choices on s_t will not be efficient at the competitive equilibrium. Intuitively, one can expect that

⁴⁴By (1), this will be satisfied when the elasticity of h_{t+1} with respect to e_t is greater than the elasticity of h_{t+1} with respect to \bar{h}_t . Indeed, as $\frac{\varepsilon_1^{\widehat{e}}}{\beta} > 1$, $\varepsilon_2^{\widehat{e}} < 0$ and ε_X^{Ω} , $\varepsilon_2^{\widehat{e}} - \frac{\varepsilon_1^{\widehat{e}}}{\beta} + \varepsilon_X^{\Omega} < 0$.

the competitive level of health expenditure will be inferior to its optimal level. However, the existence of educational inefficiency could alter this result because, as previously shown, it decreases the total cost of quantity.

5.1 The Competitive Equilibrium

Parents now have to determine the amount of health expenditure for their children. In other words, they choose X_t and s_t . The addition of an externality on health spending implies that private health investment will not be optimal. Assume that the government introduces a subsidy r_t on health expenditure in accordance with the previous fiscal system. The government budget constraint, for each date t , is now:

$$T_t = \lambda_t \theta e(h_{t+1}, h_t, h_t) \Omega(X_t) w_t h_t - \frac{\Lambda_t w_t h_t}{\xi(s_t, \bar{s}_t)} X_t + r_t s_t \quad (30)$$

When the fiscal scheme is implemented, the familial budget constraint, at date t , is:

$$C_t + (1 - r_t) s_t + \left[\frac{\sigma + \Lambda_t}{\xi(s_t, \bar{s}_t)} + \phi \right] w_t h_t X_t + (1 - \kappa_t) \theta w_t \bar{h}_t \Omega(X_t) \cdot e_t = w_t h_t \quad (31)$$

The final good can now be either consumed or invested in health. Then, a parent born in $t - 1$ determines his optimal demands $(C_t^*, X_t^*, h_{t+1}^*)$ by maximizing V_t with respect to C_t , X_t , and h_{t+1} subject to (1) and (31). As health expenditure does not enter the objective function, parents determine their optimal health expenditure by minimizing $(1 - r_t) s_t + \frac{\sigma + \Lambda_t}{\xi(s_t, \bar{s}_t)} w_t h_t X_t$. It follows that:

$$1 - r_t = \frac{[\sigma + \Lambda_t] \xi'_1(s_t^*, \bar{s}_t^*)}{[\xi(s_t^*, \bar{s}_t^*)]^2} X_t^* w_t h_t^* \quad (32)$$

Parents equalize the marginal return and the marginal cost of the health expenditure $1 - r_t$. The marginal benefit of the health expenditure (*RHS* of (32)) consists of the reduction in the total cost of quantity.⁴⁵ In other words, equation (32) determines the optimal parental spending on health to have X_t surviving children. It also emphasizes that the taxation of births increases the marginal cost of quantity and increases the marginal benefits of the health expenditure.

⁴⁵As mentionned in the Benchmark model, a higher child survival rate decreases the cost of quantity.

Using the same method as for previous models, I obtain the same first order conditions with respect to L_{t+1} and h_{t+1} as in the benchmark model given that at each date t , the child survival probability equals $\xi(s_t, \bar{s}_t)$ instead of ξ . So, the competitive equilibrium is now defined by the set $\{C_t^*, X_t^*, e_t^*, h_t^*, h_{t+1}^*, H_t^*, Y_t^*, w_t^*, s_t^*\}_{t=0}^{t=+\infty}$ satisfying equations $\{(1), (3), (4), (5), (8), (9), (31), (32)\}_{t=0}^{t=+\infty}$ given that now $\xi = \xi(s_t, \bar{s}_t)$.

5.2 The Social Optimum

For simplicity's sake, $f(L_t) = 1$ is assumed for each t . The Social Planner maximizes a Millian Social Welfare function $W_0 = \sum_{t=0}^{+\infty} \beta^t u(C_t, X_t)$. He holds a new maximization instrument s_t and faces a new resource constraint at each date t :

$$C_t + s_t = \left[1 - \left(\frac{\sigma}{\xi(s_t, \bar{s}_t)} + \phi + \theta e_t \right) X_t \right] A h_t \quad (33)$$

At the optimum $s_t = \bar{s}_t$. The Social Planner determines the optimal health expenditure by minimizing $\frac{\sigma}{\xi(s_t, \bar{s}_t)} X_t A h_t + s_t$ with regard to s_t . When this is the case, the marginal social cost of health spending (equal to one) is equal to its marginal return. Obviously, the marginal social benefit of health spending is higher than the marginal private benefit (calculated in equation (32)). Formally, the optimal decision rule for s_t is:

$$1 = \frac{\sigma \left[\hat{\xi}'_1(\hat{s}_t, \hat{s}_t) + \hat{\xi}'_2(\hat{s}_t, \hat{s}_t) \right]}{[\xi(\hat{s}_t, \hat{s}_t)]^2} \hat{X}_t \hat{A} \hat{h}_t \quad (34)$$

The optimal equilibrium now results from the maximization of W_0 with regard to L_{t+1} and h_{t+1} and subject to $\{(1), (33), (34)\}_{t=0}^{t=+\infty}$. Then the social optimum is then described by the set $\{\hat{C}_t, \hat{X}_t, \hat{h}_t, \hat{s}_t\}_{t=0}^{t=+\infty}$ satisfying equations $\{(12), (13), (33), (34)\}_{t=0}^{t=+\infty}$ given that $\xi = \xi(s_t, \bar{s}_t)$.

5.3 The Optimal Tax-Transfer Policy At The Steady State

Using the same method as in Proposition 1, it is straightforward that there exists a unique set $\{\lambda_t, \Lambda_t, s_t, T_t\}_{t=0}^{t=+\infty}$ that is able to decentralize the first-best path. An optimal policy at the steady state must make identical systems $\{(12), (13), (34)\}$ and $\{(8), (9), (32)\}$. Con-

sequently, at the steady state, the optimal fiscal scheme is:

$$\hat{\lambda} = \frac{\varepsilon_3^e}{\varepsilon_X^\Omega - \frac{\varepsilon_1^e}{\beta} - 1 - \varepsilon_2^e} > 0, \quad \frac{\hat{\Lambda}}{\xi} = \theta \Omega'_X e \frac{\varepsilon_3^e}{\varepsilon_X^\Omega - \frac{\varepsilon_1^e}{\beta} - 1 - \varepsilon_2^e} > 0, \quad \hat{r} = \frac{\xi'_2(\hat{s}, \hat{s}) - \frac{\xi'_1(\hat{s}, \hat{s}) \theta \Omega'_X e \varepsilon_3^e}{\sigma \left(\varepsilon_X^\Omega - \frac{\varepsilon_1^e}{\beta} - 1 - \varepsilon_2^e \right)}}{\xi'_1(\hat{s}, \hat{s}) + \xi'_2(\hat{s}, \hat{s})} \quad (35)$$

Optimal values of $\hat{\lambda}$ and $\hat{\Lambda}$ are the same as in the previous section (given that the optimal values of \hat{C} , \hat{X} and \hat{h} have changed). This implies that, as in Section 1, a policy of education and health is optimal when it is combined with a tax-transfer policy on births. Here, because the Social Planner exhibits no preference for the population stock, the optimal family policy always consists of a tax on births, a subsidy on education and a tax or a subsidy on health spending. The government budget constraint still has to be balanced by the implementation of a lump-sum tax on each family.

Proposition 2 *When the externality on the health expenditure is strong such that $\varepsilon_s^{\xi(s, \bar{s})} > \bar{\varepsilon}$, the optimal health policy consists of a subsidy. In the opposite case, it is optimal to tax health spending.*

Proof. It is straightforward to show that the parental health expenditure is not optimal at the competitive steady state. At the competitive steady state (without taxation), (32) and (8) imply $s^* = \varepsilon_s^{\xi(s, \bar{s})} \sigma A h N$. At the optimal steady state, (34) and (12) imply $\hat{s} = \left[\varepsilon_s^{\xi(s, \bar{s})} + \varepsilon_s^{\xi(s, \bar{s})} \right] \sigma A h N$. It follows that $s^* < \hat{s}$. However $s^* < \hat{s}$ does not ensure that health expenditure should always be subsidized. (32) and (34) indicates that the optimal value of health subsidies is:

$$\hat{r} = 1 - \frac{\varepsilon_s^{\xi(\hat{s}, \hat{s})}}{\varepsilon_s^{\xi(\hat{s}, \hat{s})} + \varepsilon_s^{\xi(\hat{s}, \hat{s})}} \left(1 - \frac{\hat{\Lambda}}{\sigma} \right)$$

Then, \hat{r} is positive if the following condition holds:

$$\varepsilon_s^{\xi(\hat{s}, \hat{s})} > - \frac{\theta \Omega'_X e \varepsilon_s^{\xi(\hat{s}, \hat{s})}}{\sigma \left(\varepsilon_X^\Omega - \frac{\varepsilon_1^e}{\beta} - 1 - \varepsilon_2^e \right)} \cdot \varepsilon_3^e \equiv \bar{\varepsilon}$$

■

When health externalities are strong with respect to educational externalities ($\varepsilon_s^{\xi(\hat{s}, \hat{s})} > \bar{\varepsilon}$), the health expenditure has to be subsidized while the opposite is true when ($\varepsilon_s^{\xi(\hat{s}, \hat{s})} < \bar{\varepsilon}$). This result comes from the non-linearity of the costs structure. Indeed, the existence of an

externality on the health expenditure implies that parents do not internalize all the returns on their investment in children's health. The comparison of (32) with $\Lambda = r = 0$ and (34) indicates that the health expenditure at the competitive steady state is lower than at the optimal steady state. However, when education is subsidized, a tax on births has to be implemented. When this is the case, the cost of quantity is increased relative to the cost of health, so parents tend to increase their health expenditure. The tax on births plays the role of an indirect subsidy on health. Finally, the sign of \hat{r} is determined by the difference between the intensity of externalities on both health and education investments. If the externality on health is relatively strong ($\varepsilon_{\hat{s}}^{\xi(\hat{s}, \hat{s})} > \bar{\varepsilon}$), the indirect subsidy will not be sufficient to reach \hat{s} , so \hat{r} will be positive. Conversely, if the externality on health is relatively weak ($\varepsilon_{\hat{s}}^{\xi(\hat{s}, \hat{s})} < \bar{\varepsilon}$), the indirect subsidy exceeds the necessary health subsidy. Therefore, \hat{r} will be negative meaning that health expenditure have to be taxed.

To summarize, the present section provides two results. First, in a Millian economy, whenever it is optimal to subsidize education *and* health, it is optimal to tax births. Second, when the social returns on the health expenditure are low relative to the social return on education expenditure, the optimal family planning program consists of the promotion of education financed by the taxation of health and births and a lump-sum tax. Conversely, when the social returns on health expenditure are high relative to social returns on education, the optimal family planning program consists in the promotion of education and health financed by the taxation of births and a lump-sum tax.⁴⁶ This optimal policy has, in fact, two main objectives. The first is to modify the parental trade-off between quality and quantity by inciting parents to transfer a part of their spending on fertility toward educational investment. The second objective is to modify the parental trade-off between fertility and health. In order to reach the same number of surviving children, parents are incited to invest less in the quantity of children.

⁴⁶Following Dasgupta [1993], the social returns on health expenditure are high. Then, the promotion of education and health financed by the taxation of births is a more realistic conclusion.

6 Some Empirical Issues At Stake

In this section, I discuss the main theoretical conclusions of the model in the light of some empirical evidence. I show that these conclusions could enrich the set of family policies that are implemented in countries facing the problem of overpopulation. It should be clear that the simplicity of the model does not allow it to reproduce the very complex demographic puzzles that confront these countries. The discussion is therefore limited to general statements.

Countries that face over-population problems implement policies to slow their population growth rate. Two examples are particularly illuminating: China and Sub-Saharan Africa. Although these two regions both face overpopulation, their family policies have been noticeably different. This section reflects on improvements that could be made to these countries' policies in light of the fiscal scheme proposed in this paper.

A recent report from the World Bank [2007] states that 31 of the 35 countries with the highest fertility rates are located in Sub-Saharan Africa. For the majority of these countries, fertility rates remained stable over the last few decades at greater than six children per woman. However, the vast majority of these countries have implemented family planning programs in collaboration with international organizations such as the World Bank.

The World Bank's report [2007] emphasizes that the main factor driving high fertility rates is the persistent parents desire for a large number of children. In other words, the too high fertility rates in Sub-Saharan Africa do not result from a lack of family planning programs. The report asserts that efforts must be made to reduce the desired fertility. To do so, it recommends improving education and health programs at the local level. However, education indicators have increased since the sixties. Between 1990 and 2006, the net primary school enrollment rate increased from 50 to 70 percent. In the same period, the youth and adult literacy rates increased as well.⁴⁷ This noticeable improvement in education rates has not been sufficient to reduce fertility rates.

The present paper does not recommend increasing spending on family planning programs. It proposes complementing family planning programs with taxes on births that would finance

⁴⁷In Sub-Saharan Africa, the youth literacy rate was 64% in 1990 and 73% in 2006. The adult literacy rate was 54% in 1990 and 61% in 2006. See Appendix E for a more complete description.

education and health. Without taxing births, these programs reduce the net cost of the children's quantity, creating conditions for the number of children desired to remain high.

Obviously, it is unclear whether it is feasible to implement a tax on births in a population that is largely engaged in an informal economy.⁴⁸ However, increasing the costs associated with increases in the quantity of children should be considered as a policy instrument for family planning.

China also implements family policy to reduce its population growth rate. However, its strategy differs from family planning programs in Sub-Saharan Africa. Since 1980, China has implemented the "One-Child" policy which strongly constrains families' fertility. It is a system that provides generous subsidies for the first birth and imposes very high taxes on the subsequent births. If parents decide to have a second child without being permitted to do so, they lose a large part of their retirement pension, their child care allowance and other social advantages. Furthermore, some physical sanctions have been implemented in rural areas. This fiscal scheme is different from the one proposed in this paper, as the Chinese policy does not tax all births at the same rate. The first birth is subsidized while subsequent births are heavily taxed.

The high tax on subsequent births is a very efficient incentive to have only one child. Thus, the large majority of families are subsidized to reach the target of one child per family making the "One-Child" Policy very costly. The policy does not produce revenue to finance education and health policies, so there is nothing to ensure that the relative costs of education and health will reach their optimal value. Indeed, a large body literature stresses the insufficiency of the public expenditure on health and education in Chinese rural areas where the large majority of the population is concentrated (for example, see Kanbur & Zhang [2003] and Fan & Zhang [2000]).

The results of this paper indicate that some marginal changes in the One-Child policy could improve the overall efficiency of Chinese family planning policy. These results suggest that all births should be taxed to avoid effective costs. The amount saved by the Chinese government could be invested in more ambitious education and health policies, thereby

⁴⁸Furthermore, some of these countries are facing other complex problems such as political instability, starvation and HIV pandemy that are well beyond the scope of this paper. These problems have a direct and significant effect on fertility and education behaviors.

reducing the large inequalities between urban and rural areas. *Theoretically*, this system would not increase the overall cost of the Chinese family planning program and would lead to the same fertility rates. It would, however, increase health and education investments. Furthermore, the Chinese family policy is coercive, while the economic policy proposed in this paper is non-coercive. If the two policies are equally efficient, the non-coercive policy should be implemented because it improves welfare.

Note that a "naive" interpretation of the model could lead to an alternative analysis. The Chinese government's objective to reach a fertility rate inferior to two children per family could reveal that the Chinese optimal population growth rate in the long-run is negative and that there exists a preference for large generations. Then the policy of low subsidies on education spending would be optimal because it transfers welfare from future to present generations.

7 Conclusion

The present paper analyses optimal family policies in the standard model of trade-off between quality and quantity. Given the non-linearity of the parental budget constraint, subsidizing education and health will be optimal if a tax (or a subsidy) on births is also implemented. Indeed, a subsidy on education reduces both the cost of educational investment and the total cost of fertility. This result applies for a large set of Social Welfare Functions, including Millian and Benthamite functions. Obviously, the model concludes that taxing births without financing education and health is also not optimal.

Finally, the fiscal scheme proposed in this model is quite simple: education and health expenditures are promoted by the taxation of births and lump-sum transfers. This scheme could improve the overall efficiency of family policies currently implemented in China and Sub-Saharan Africa. The main objective of the present investigation was to explore the family policy recommendations of the standard endogenous fertility model. As a natural extension of this work, future research should integrate countries' specificities to make quantitative proposals for economic policy and to provide a more precise discussion of empirical evidence.

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Appendix A

Analyzing sub-systems $\{(12), (13)\}_{t=0}^{t=+\infty}$ to $\{(16), (17)\}_{t=0}^{t=+\infty}$, it appears that the set $\{\lambda_t, \Lambda_t\}_{t=0}^{t=+\infty}$ has to ensure that, at each date t , the first order conditions with respect to L_{t+1} and h_{t+1} at the competitive equilibrium is identical to the first order condition with respect to L_{t+1} and h_{t+1} at the optimum. Doing so, I obtain the following system of equations, given the initial conditions $(L_0, h_0) > (0, 0)$ and that $\forall t, X_t^* = \hat{X}_t$ and $h_t^* = \hat{h}_t$:

$$t = 0$$

$$\frac{\hat{X}_0}{\hat{X}_1} \frac{u'_{X_0} - A\hat{h}_0 \left(\frac{\sigma + \Lambda_0}{\xi} + \phi + (1 - \lambda_0)\theta\Omega'_{X_0} e[\hat{h}_1, \cdot, \cdot] \right) u'_{C_0}}{A\hat{h}_1 \left(\frac{\sigma + \Lambda_1}{\xi} + \phi + (1 - \lambda_1)\theta\Omega'_{X_1} e[\hat{h}_2, \cdot, \cdot] \right) u'_{C_1} - u'_{X_1}} = \frac{u'_{X_0} - A\hat{h}_0 \left(\frac{\sigma}{\xi} + \phi + \theta\Omega'_{X_0} e[\hat{h}_1, \cdot, \cdot] \right) u'_{C_0}}{\varepsilon_L^f u(\cdot, \hat{X}_1) + F(\hat{X}_0) \frac{\hat{X}_1}{\hat{X}_0} \left[A\hat{h}_1 \left(\frac{\sigma}{\xi} + \phi + \theta\Omega'_{X_1} e[\hat{h}_2, \cdot, \cdot] \right) u'_{C_1} - u'_{X_1} \right]} \quad (36)$$

$$\frac{F(\hat{X}_0) \left[1 - \left(\frac{\sigma}{\xi} + \phi \right) \hat{X}_1 - \theta\Omega(\hat{X}_1) (e[\hat{h}_2, \cdot, \cdot] + \hat{h}_1 (e'_2[\hat{h}_2, \cdot, \cdot] + e'_3[\hat{h}_2, \cdot, \cdot])) \right]}{1 - \left(\frac{\sigma + \Lambda_1}{\xi} + \phi \right) X_1 - (1 - \lambda_1)\theta\Omega(X_1) (e[h_2, \cdot, \cdot] + h_1 e'_2[h_2, \cdot, \cdot])} = \frac{1}{1 - \lambda_0} \quad (37)$$

$t = 1$

$$\frac{\widehat{X}_1}{\widehat{X}_2} \frac{u'_{X_1} - A\widehat{h}_1\left(\frac{\sigma+\Lambda_1}{\xi} + \phi + (1-\lambda_1)\theta\Omega'_{X_1} e[\widehat{h}_2, \cdot, \cdot]\right) u'_{C_1}}{A\widehat{h}_2\left(\frac{\sigma+\Lambda_2}{\xi} + \phi + (1-\lambda_2)\theta\Omega'_{X_2} e[\widehat{h}_3, \cdot, \cdot]\right) u'_{C_2} - u'_{X_2}} = \frac{u'_{X_1} - A\widehat{h}_1\left(\frac{\sigma}{\xi} + \phi + \theta\Omega'_{X_1} e[\widehat{h}_2, \cdot, \cdot]\right) u'_{C_1}}{\varepsilon_L^f u(\cdot, \widehat{X}_2) + F(\widehat{X}_1) \frac{\widehat{X}_2}{\widehat{X}_1} \left[A\widehat{h}_2\left(\frac{\sigma}{\xi} + \phi + \theta\Omega'_{X_2} e[\widehat{h}_3, \cdot, \cdot]\right) u'_{C_2} - u'_{X_2} \right]} \quad (38)$$

$$\frac{F(\widehat{X}_1) \left[1 - \left(\frac{\sigma}{\xi} + \phi \right) \widehat{X}_2 - \theta\Omega(\widehat{X}_2) \left(e[\widehat{h}_3, \cdot, \cdot] + \widehat{h}_2(e'_2[\widehat{h}_3, \cdot, \cdot] + e'_3[\widehat{h}_3, \cdot, \cdot]) \right) \right]}{1 - \left(\frac{\sigma+\Lambda_2}{\xi} + \phi \right) X_2 - (1-\lambda_2)\theta\Omega(X_2) \left(e[h_3, \cdot, \cdot] + h_2 e'_2[h_3, \cdot, \cdot] \right)} = \frac{1}{1-\lambda_1} \quad (39)$$

...

...

$t = s$

$$\begin{aligned} \frac{\widehat{X}_s}{\widehat{X}_{s+1}} \frac{u'_{X_s} - A\widehat{h}_s\left(\frac{\sigma+\Lambda_s}{\xi} + \phi + (1-\lambda_s)\theta\Omega'_{X_s} e[\widehat{h}_{s+1}, \cdot, \cdot]\right) u'_{C_s}}{A\widehat{h}_{s+1}\left(\frac{\sigma+\Lambda_{s+1}}{\xi} + \phi + (1-\lambda_{s+1})\theta\Omega'_{X_{s+1}} e[\widehat{h}_{s+2}, \cdot, \cdot]\right) u'_{C_{s+1}} - u'_{X_{s+1}}} = \\ \frac{u'_{X_s} - A\widehat{h}_s\left(\frac{\sigma}{\xi} + \phi + \theta\Omega'_{X_s} e[\widehat{h}_{s+1}, \cdot, \cdot]\right) u'_{C_s}}{\varepsilon_L^f u(\cdot, \widehat{X}_{s+1}) + F(\widehat{X}_s) \frac{\widehat{X}_{s+1}}{\widehat{X}_s} \left[A\widehat{h}_{s+1}\left(\frac{\sigma}{\xi} + \phi + \theta\Omega'_{X_{s+1}} e[\widehat{h}_{s+2}, \cdot, \cdot]\right) u'_{C_{s+1}} - u'_{X_{s+1}} \right]} \\ \frac{F(\widehat{X}_s) \left[1 - \left(\frac{\sigma}{\xi} + \phi \right) \widehat{X}_{s+1} - \theta\Omega(\widehat{X}_{s+1}) \left(e[\widehat{h}_{s+2}, \cdot, \cdot] + \widehat{h}_{s+1}(e'_2[\widehat{h}_{s+2}, \cdot, \cdot] + e'_3[\widehat{h}_{s+2}, \cdot, \cdot]) \right) \right]}{1 - \left(\frac{\sigma+\Lambda_{s+1}}{\xi} + \phi \right) X_{s+1} - (1-\lambda_{s+1})\theta\Omega(X_{s+1}) \left(e[h_{s+2}, \cdot, \cdot] + h_{s+1} e'_2[h_{s+2}, \cdot, \cdot] \right)} = \frac{1}{1-\lambda_s} \end{aligned} \quad (40)$$

$t = s + 1$

$$\begin{aligned} \frac{\widehat{X}_{s+1}}{\widehat{X}_{s+2}} \frac{u'_{X_{s+1}} - A\widehat{h}_{s+1}\left(\frac{\sigma+\Lambda_{s+1}}{\xi} + \phi + (1-\lambda_{s+1})\theta\Omega'_{X_{s+1}} e[\widehat{h}_{s+2}, \cdot, \cdot]\right) u'_{C_{s+1}}}{A\widehat{h}_{s+2}\left(\frac{\sigma+\Lambda_{s+2}}{\xi} + \phi + (1-\lambda_{s+2})\theta\Omega'_{X_{s+2}} e[\widehat{h}_{s+3}, \cdot, \cdot]\right) u'_{C_{s+2}} - u'_{X_{s+2}}} = \\ \frac{(u'_{X_{s+1}} - A\widehat{h}_{s+1}\left(\frac{\sigma}{\xi} + \phi + \theta\Omega'_{X_{s+1}} e[\widehat{h}_{s+2}, \cdot, \cdot]\right) u'_{C_{s+1}})}{\varepsilon_L^f u(\cdot, \widehat{X}_{s+2}) + F(\widehat{X}_{s+1}) \frac{\widehat{X}_{s+2}}{\widehat{X}_{s+1}} \left[A\widehat{h}_{s+2}\left(\frac{\sigma}{\xi} + \phi + \theta\Omega'_{X_{s+2}} e[\widehat{h}_{s+3}, \cdot, \cdot]\right) u'_{C_{s+2}} - u'_{X_{s+2}} \right]} \\ \frac{F(\widehat{X}_{s+1}) \left[1 - \left(\frac{\sigma}{\xi} + \phi \right) \widehat{X}_{s+2} - \theta\Omega(\widehat{X}_{s+2}) \left(e[\widehat{h}_{s+3}, \cdot, \cdot] + \widehat{h}_{s+2}(e'_2[\widehat{h}_{s+3}, \cdot, \cdot] + e'_3[\widehat{h}_{s+3}, \cdot, \cdot]) \right) \right]}{1 - \left(\frac{\sigma+\Lambda_{s+2}}{\xi} + \phi \right) X_{s+2} - (1-\lambda_{s+2})\theta\Omega(X_{s+2}) \left(e[h_{s+3}, \cdot, \cdot] + h_{s+2} e'_2[h_{s+3}, \cdot, \cdot] \right)} = \frac{1}{1-\lambda_{s+1}} \end{aligned} \quad (41)$$

...

Equations (36) and (37) characterize a system of 2 equations with 2 unknowns which are $\{\lambda_0, \Lambda_0\}$ given the parental perfect foresight on $\{\lambda_1, \Lambda_1\}$. This system is linear with regards to its unknowns and so, it is straightforward that there exists a unique solution to this system of equations. I can display $\{\lambda_0, \Lambda_0\}$ as a linear function of λ_1 and Λ_1 . It is intuitive that the assumption of perfect foresights does not alter my result because, for any

parental expectation on $\{\lambda_1, \Lambda_1\}$, there exist a solution to the sub-system of equations. This reasoning can be applied at each date t .

So, I can determine $\hat{\lambda}_t$ and $\hat{\Lambda}_t$ as the values of λ_t and Λ_t making identical equation (16) to equation (12) and equation (17) to (13) for each date t . I obtain the following result:

$$\begin{aligned}\hat{\lambda}_t &= -a_t \hat{\lambda}_{t+1} + b_t \hat{\Lambda}_{t+1} + d_t [F(X_t) - 1] - g_t \\ \hat{\Lambda}_t &= -i_t \hat{\lambda}_{t+1} + j_t \hat{\Lambda}_{t+1} - k_t [F(X_t) - 1] - m_t\end{aligned}$$

with

$$\begin{aligned}a_t &\equiv \frac{\left(1 + \varepsilon_2^{e(h_{t+2}, \cdot, \cdot)}\right) \theta \Omega(X_{t+1}) e(h_{t+2}, \cdot, \cdot)}{F(X_t) \left[\frac{C_{t+1}}{Ah_{t+1}} - \theta \Omega(X_{t+1}) e(h_{t+2}, \cdot, \cdot) \left(\varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} + \varepsilon_3^{e(h_{t+2}, \cdot, \cdot)} \right) \right]} \\ b_t &\equiv \frac{X_{t+1}}{F(X_t) \left[\frac{C_{t+1}}{Ah_{t+1}} - \theta \Omega(X_{t+1}) e(h_{t+2}, \cdot, \cdot) \left(\varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} + \varepsilon_3^{e(h_{t+2}, \cdot, \cdot)} \right) \right]} \\ d_t &\equiv \frac{\frac{C_{t+1}}{Ah_{t+1}}}{F(X_t) \left[\frac{C_{t+1}}{Ah_{t+1}} - \theta \Omega(X_{t+1}) e(h_{t+2}, \cdot, \cdot) \left(\varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} + \varepsilon_3^{e(h_{t+2}, \cdot, \cdot)} \right) \right]} \\ g_t &\equiv \frac{\theta \Omega(X_t) e(h_{t+2}, \cdot, \cdot) \left[\varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} + \varepsilon_3^{e(h_{t+2}, \cdot, \cdot)} \right]}{\frac{C_{t+1}}{Ah_{t+1}} - \theta \Omega(X_{t+1}) e(h_{t+2}, \cdot, \cdot) \left(\varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} + \varepsilon_3^{e(h_{t+2}, \cdot, \cdot)} \right)} \\ i_t &\equiv \theta \Omega(X_{t+1}) e(h_{t+2}, \cdot, \cdot) \left[\frac{\left(1 + \varepsilon_2^{e(h_{t+2}, \cdot, \cdot)}\right) \theta e(h_{t+1}, \cdot, \cdot) \Omega'_{X_t}}{F(X_t) \left[\frac{C_{t+1}}{Ah_{t+1}} - \theta \Omega(X_{t+1}) e(h_{t+2}, \cdot, \cdot) \left(\varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} + \varepsilon_3^{e(h_{t+2}, \cdot, \cdot)} \right) \right]} + \beta \frac{\varepsilon_{X_{t+1}}^\Omega u'_{C_{t+1}} h_{t+1}}{u'_{C_t} h_t X_t} \right] \\ j_t &\equiv \frac{\theta e(h_{t+1}, \cdot, \cdot) \Omega'_{X_t} X_{t+1}}{F(X_t) \left[\frac{C_{t+1}}{Ah_{t+1}} - \theta \Omega(X_{t+1}) e(h_{t+2}, \cdot, \cdot) \left(\varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} + \varepsilon_3^{e(h_{t+2}, \cdot, \cdot)} \right) \right]} + \beta \frac{u'_{C_{t+1}} h_{t+1} X_{t+1}}{u'_{C_t} h_t X_t} \\ k_t &\equiv \frac{X_{t+1}}{X_t} \frac{\theta \Omega(X_t) e(h_{t+1}, \cdot, \cdot) \left[\varepsilon_{X_t}^\Omega \frac{C_{t+1}}{Ah_{t+1}} + \varepsilon_1^{e(h_{t+1}, \cdot, \cdot)} \left(\frac{\sigma}{\xi} + \phi + \Omega'_{X_{t+1}} \theta e(h_{t+2}, \cdot, \cdot) - \frac{u'_{X_{t+1}}}{Ah_{t+1}} \right) \right]}{F(X_t) \left[\frac{C_{t+1}}{Ah_{t+1}} - \theta \Omega(X_{t+1}) e(h_{t+2}, \cdot, \cdot) \left(\varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} + \varepsilon_3^{e(h_{t+2}, \cdot, \cdot)} \right) \right]} \\ m_t &\equiv \frac{\theta \Omega(X_t) e(h_{t+2}, \cdot, \cdot) \left[\varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} + \varepsilon_3^{e(h_{t+2}, \cdot, \cdot)} \right] e(h_{t+1}, \cdot, \cdot) \Omega'_{X_t}}{\frac{C_{t+1}}{Ah_{t+1}} - \theta \Omega(X_{t+1}) e(h_{t+2}, \cdot, \cdot) \left(\varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} + \varepsilon_3^{e(h_{t+2}, \cdot, \cdot)} \right)} + \beta \frac{\varepsilon_L^f u(\cdot, X_{t+1}) X_{t+1}}{Au'_{C_t} X_t h_t}\end{aligned}$$

Appendix B

Using the same method as in the Benchmark model, the economic policy that decentralizes the first-best path is the set $\{\lambda_t, \Lambda_t\}_{t=0}^{t=+\infty}$ solving the following system of equations at

each date t :

$$\frac{\varepsilon_L^f X_t^{1-\varepsilon} U(C_{t+1}) + F(\widehat{X}_t) \frac{\widehat{X}_{t+1}}{\widehat{X}_t} \left[A\widehat{h}_{t+1} \left(\frac{\sigma}{\xi} + \phi + \theta \Omega'_{X_{t+1}} e[\widehat{h}_{t+2}, \cdot, \cdot] \right) X_{t+1}^{1-\varepsilon} U'_{C_{t+1}} - (1-\varepsilon) X_{t+1}^{-\varepsilon} U(C_{t+1}) \right]}{(1-\varepsilon) X_t^{-\varepsilon} U(C_t) - Ah_t \left(\frac{\sigma}{\xi} + \phi + \theta \Omega'_{X_t} e[\widehat{h}_{t+1}, \cdot, \cdot] \right) X_t^{1-\varepsilon} U'_{C_t}} = \frac{\widehat{X}_{t+1}}{\widehat{X}_t} \frac{Ah_{t+1} \left(\frac{\sigma + \Lambda_{t+1}}{\xi} + \phi + (1-\lambda_{t+1}) \theta \Omega'_{X_{t+1}} e[h_{t+2}, \cdot, \cdot] \right) X_{t+1}^{1-\varepsilon} U'_{C_{t+1}} - (1-\varepsilon) X_{t+1}^{-\varepsilon} U(C_{t+1})}{(1-\varepsilon) X_t^{-\varepsilon} U(C_t) - Ah_t \left(\frac{\sigma + \Lambda_t}{\xi} + \phi + (1-\lambda_t) \theta \Omega'_{X_t} e[h_{t+1}, \cdot, \cdot] \right) X_t^{1-\varepsilon} U'_{C_t}} \quad (42)$$

$$\frac{\theta h_t \Omega(X_t) e'_1[\widehat{h}_{t+1}, \cdot, \cdot]}{\beta F(\widehat{X}_t) \left[1 - \left(\frac{\sigma}{\xi} + \phi \right) \widehat{X}_{t+1} - \theta \Omega(\widehat{X}_{t+1}) \left(e[\widehat{h}_{t+2}, \cdot, \cdot] + \widehat{h}_{t+1} (e'_2[\widehat{h}_{t+2}, \cdot, \cdot] + e'_3[\widehat{h}_{t+2}, \cdot, \cdot]) \right) \right]} = \frac{(1-\lambda_t) \theta h_t \Omega(X_t) e'_1[h_{t+1}, \bar{h}_t]}{\beta \left[1 - \left(\frac{\sigma + \Lambda_{t+1}}{\xi} + \phi \right) X_{t+1} - (1-\lambda_{t+1}) \theta \Omega(X_{t+1}) \left(e[h_{t+2}, h_{t+1}, \bar{h}_{t+1}] + h_{t+1} e'_2[h_{t+2}, h_{t+1}, \bar{h}_{t+1}] \right) \right]} \quad (43)$$

Given the parental (perfect) expectations on $\{\lambda_{t+1}, \Lambda_{t+1}\}_{t=0}^{t=+\infty}$, there exists a unique vector $\{\lambda_t, \Lambda_t\}_{t=0}^{t=+\infty}$ that is able to decentralize the first-best path. Given that \widehat{T}_t is directly deduced from the government budget constraint, the optimal economic policy is fully described as follows:

$$\begin{aligned} \widehat{\lambda}_t &= -a_t \widehat{\lambda}_{t+1} + b_t \widehat{\Lambda}_{t+1} + d_t [F(X_t) - 1] - g_t \\ \widehat{\Lambda}_t &= -\widetilde{i}_t \widehat{\lambda}_{t+1} + \widetilde{j}_t \widehat{\Lambda}_{t+1} - \widetilde{k}_t [F(X_t) - 1] - \widetilde{m}_t \end{aligned}$$

with:

$$\begin{aligned} \widetilde{i}_t &\equiv \theta \Omega(X_{t+1}) e(h_{t+2}, \cdot, \cdot) \\ &\times \left[\frac{\left(1 + \varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} \right) \theta e(h_{t+1}, \cdot, \cdot) \Omega'_{X_t}}{F(X_t) \left[\frac{C_{t+1}}{Ah_{t+1}} - \theta \Omega(X_{t+1}) e(h_{t+2}, \cdot, \cdot) \left(\varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} + \varepsilon_3^{e(h_{t+2}, \cdot, \cdot)} \right) \right]} + \beta \frac{U'_{C_t} \varepsilon_{X_{t+1}}^{h_{t+1}}}{U'_{C_{t+1}} h_t X_t} \left(\frac{X_{t+1}}{X_t} \right)^{1-\varepsilon} \right] \\ \widetilde{j}_t &\equiv \frac{\theta e(h_{t+1}, \cdot, \cdot) \Omega'_{X_t} X_{t+1}}{F(X_t) \left[\frac{C_{t+1}}{Ah_{t+1}} - \theta \Omega(X_{t+1}) e(h_{t+2}, \cdot, \cdot) \left(\varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} + \varepsilon_3^{e(h_{t+2}, \cdot, \cdot)} \right) \right]} + \beta \frac{h_{t+1} X_{t+1}}{h_t X_t} \left(\frac{X_{t+1}}{X_t} \right)^{1-\varepsilon} \frac{U'_{C_t}}{U'_{C_{t+1}}} \\ \widetilde{k}_t &\equiv \frac{X_{t+1}}{X_t} \frac{\theta \Omega(X_t) e(h_{t+1}, \cdot, \cdot) \left[\varepsilon_{X_t}^{\Omega} \frac{C_{t+1}}{Ah_{t+1}} + \varepsilon_1^{e(h_{t+1}, \cdot, \cdot)} \left(\frac{\sigma}{\xi} + \phi + \theta \Omega'_{X_{t+1}} \theta e(h_{t+2}, \cdot, \cdot) - \frac{(1-\varepsilon) X_{t+1}^{-\varepsilon} U(C_{t+1})}{Ah_{t+1}} \right) \right]}{F(X_t) \left[\frac{C_{t+1}}{Ah_{t+1}} - \theta \Omega(X_{t+1}) e(h_{t+2}, \cdot, \cdot) \left(\varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} + \varepsilon_3^{e(h_{t+2}, \cdot, \cdot)} \right) \right]} \\ \widetilde{m}_t &\equiv \frac{\theta \Omega(X_t) e(h_{t+2}, \cdot, \cdot) \left[\varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} + \varepsilon_3^{e(h_{t+2}, \cdot, \cdot)} \right] e(h_{t+1}, \cdot, \cdot) \Omega'_{X_t}}{\frac{C_{t+1}}{Ah_{t+1}} - \theta \Omega(X_{t+1}) e(h_{t+2}, \cdot, \cdot) \left(\varepsilon_2^{e(h_{t+2}, \cdot, \cdot)} + \varepsilon_3^{e(h_{t+2}, \cdot, \cdot)} \right)} + \beta \frac{\varepsilon_L^f U(C_{t+1}) X_{t+1}}{AU'_{C_{t+1}} X_t h_t} \end{aligned}$$

The optimal values of the instruments at the steady state are:

$$\hat{\lambda}_\infty = \frac{-F(X)\varepsilon_3^e + \frac{\beta\varepsilon_L^f X}{Ah\Omega(X)\theta e(1-\beta F(X))} \frac{U}{U'_C} - [1-F(X)] \left[\frac{C}{Ah\Omega(X)\theta e} - \varepsilon_2^e \right]}{1 + \frac{\varepsilon_1^e}{\beta} + \varepsilon_2^e - \varepsilon_X^\Omega} \quad (44)$$

$$\frac{\hat{\Lambda}}{\hat{\xi}_\infty} = \frac{\frac{\beta\varepsilon_L^f}{[1-\beta F(X)]Ah} \frac{U}{U'_C} \frac{1 + \frac{\varepsilon_1^e}{\beta} + \varepsilon_2^e}{1 + \frac{\varepsilon_1^e}{\beta} + \varepsilon_2^e - \varepsilon_X^\Omega}}{\frac{\varepsilon_1^e}{1 + \frac{\varepsilon_1^e}{\beta} + \varepsilon_2^e - \varepsilon_X^\Omega}} - \frac{\Omega'_X \theta e \left[F(X)\varepsilon_3^e + (1-F(X)) \left(\frac{C}{Ah} - \Omega(X)\theta h e'_2 \right) \right]}{1 + \frac{\varepsilon_1^e}{\beta} + \varepsilon_2^e - \varepsilon_X^\Omega} \quad (45)$$

Appendix C

The competitive equilibrium is described by the set $\{C^*, X^*, e^*, h^*, H^*, Y^*, w^*\}$ satisfying equations (1), (2), (3), (4), (5) and the following First Order Conditions:

$$\frac{U'_X}{U'_C} = \left(\frac{\sigma + \Lambda}{\xi} + \phi + \Omega'(X^*) \theta (1 - \lambda) e(h^*, h^*, h^*) \right) Ah^* \quad (46)$$

$$\frac{U'_h}{U'_C} = \theta Ah^* \Omega(X^*) (1 - \lambda) e'_1(h^*, h^*, h^*) \quad (47)$$

The optimal steady state is described by the set $\{\hat{C}, \hat{X}, \hat{h}\}$ satisfying equation (11) and the following First Order Conditions:

$$\frac{U'_X}{U'_C} = \left(\frac{\sigma}{\xi} + \phi + \theta \Omega'(\hat{X}) e(\hat{h}, \hat{h}, \hat{h}) \right) A \hat{h} U'_C \quad (48)$$

$$\frac{U'_{h_{t+1}}}{U'_C} = A \left(\hat{X} \left[\frac{\sigma}{\xi} + \phi \right] + \theta \Omega(\hat{X}) \left[e(\hat{h}, \hat{h}, \hat{h}) + \hat{h}(\hat{e}'_1 + \hat{e}'_2 + \hat{e}'_3) \right] - 1 \right) \quad (49)$$

Then, the optimal economic policy makes identical systems $\{(46), (47)\}$ and $\{(48), (49)\}$. The solution of this system of equations is displayed in (26).

Appendix D

Given that, at the equilibrium $h_t = \bar{h}_t \forall t$, the social optimum is the same as in the Benchmark model. However, the competitive equilibrium is now described by the set $\{C_t^*, X_t^*, e_t^*, h_t^*, h_{t+1}^*, H_t^*, Y_t^*, w_t^*\}_{t=0}^{t=+\infty}$ satisfying equations $\{(1), (3), (5), (27), (28)\}_{t=0}^{t=+\infty}$ and

the two following first order conditions with respect to L_{t+1} and h_{t+1} for each date t :

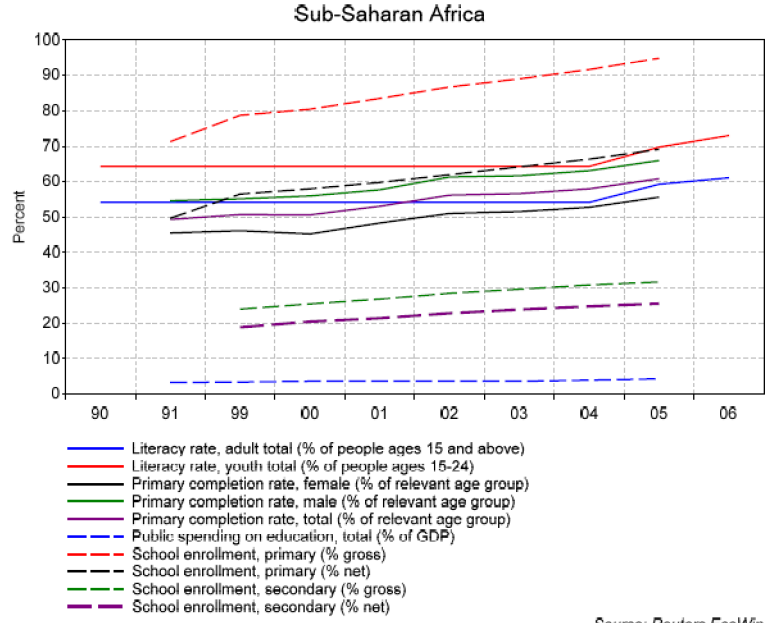
$$-\beta \frac{X_{t+1}}{X_t} = \frac{u'_{X_t} - Ah_t \left(\frac{\sigma + \Lambda_t}{\xi} + \phi + (1 - \lambda_t) \theta \Omega'_{X_t} e[h_{t+1}, \cdot, \cdot] \right) u'_{C_t}}{Ah_{t+1} \left(\frac{\sigma + \Lambda_{t+1}}{\xi} + \phi + (1 - \lambda_{t+1}) \theta \Omega'_{X_{t+1}} e[h_{t+2}, \cdot, \cdot] \right) u'_{C_{t+1}} - u'_{X_{t+1}}}$$

$$\frac{u'_{C_{t+1}}}{u'_{C_t}} = \frac{(1 - \lambda_t) \theta h_t \Omega(X_t) e'_1[h_{t+1}, h_t, \bar{h}_t]}{\beta \left[1 - \left(\frac{\sigma + \Lambda_{t+1}}{\xi} + \phi \right) X_{t+1} - (1 - \lambda_{t+1}) \theta \Omega(X_{t+1}) \bar{h}_{t+1} e'_2[h_{t+2}, h_{t+1}, \bar{h}_{t+1}] \right]}$$

Finally, changes appear only in the first order condition with respect to h_{t+1} . Using the same method as in the Benchmark model, it is straightforward that there exists a unique vector $\{\lambda_t, \Lambda_t\}_{t=0}^{t=+\infty}$ that is able to decentralize the first-best. After some straightforward calculus, I can display its stationary values:

$$\hat{\kappa} = \frac{1 + \varepsilon_3^{\hat{e}}}{\varepsilon_2^{\hat{e}} - \frac{\varepsilon_1^{\hat{e}}}{\beta} + \varepsilon_{\hat{X}}^{\Omega}} \quad \hat{\frac{\Lambda}{\xi}} = \Omega'_{\hat{X}} \theta \hat{e} \frac{1 + \varepsilon_3^{\hat{e}}}{\varepsilon_2^{\hat{e}} - \frac{\varepsilon_1^{\hat{e}}}{\beta} + \varepsilon_{\hat{X}}^{\Omega}}$$

Appendix E



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